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Analysis of arbitrary composite sections in biaxial bending and axial load

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ABSTRACT

A new methodology is presented for the ultimate strength and moment–curvature analysis of arbitrary composite sections under biaxial bending and axial load. The definition of section geometry and material properties can be unconditionally complex, based on an object-oriented implementation. Stress integration is performed using a Green path integral, with an adaptive strain-mapped Gaussian sampling. Derivative-free solution strategies for the calculation of incremental and ultimate response are applied. Results are presented in the form of moment–curvature curves, ultimate strength interaction curves and 3D failure surfaces. The performance of the methodology is demonstrated through various case studies, comparisons and benchmarks.

1. Introduction

Reinforced concrete and composite structural elements, widely used in buildings and bridges, are generally subjected to a combined action of biaxial bending and axial load. This is a result of their section geometry and material composition, their position and orientation in the structure and, most importantly, the nature of external loading. Columns at corners or under two-way slabs, bridge piers and composite decks, subjected to wind or earthquake excitation, are representative cases. Assessing the adequacy of these sections at ultimate limit state (usually via interaction diagrams) or providing information on their inelastic response gradually up to failure (in the form of moment–curvature curves), is a computationally intense task, mainly due to material nonlinearities and geometrical complexities, particularly for composite sections. Therefore, addressing this problem requires efficient solution algorithms, characterized not only by the requisite robustness but also by execution speed, since section analysis is often a repetitive task [1] within the broader computational frameworks of nonlinear structural analysis, design, assessment and retrofit.

The problem of arbitrary section analysis has received attention in the literature since the early 1960's (e.g. [2–4]), however this attention has been intensified during the last decade along with the advent of inexpensive personal computers. Various analytical, numerical and mixed methodologies for analyzing sections of varying complexity in terms of geometry and material composition have been suggested, with varying degrees of reported efficiency in terms of convergence stability and speed. Since the literature is extensive, a proper categorization is attempted hereinafter, in order to assess whether the research field under discussion is open for further novelties and improvements. In this direction, five key characteristics of section analysis methodologies have been identified, namely (a) the type of arbitrary section addressed, i.e. whether it is reinforced concrete (R/C) or generally composite, (b) the form of material constitutive laws, e.g. polynomial or arbitrary functions, (c) whether section subdivisions are imposed prior to stress integration, (d) the stress integration scheme and (e) the solution strategies for applying force equilibrium conditions.

A non-exhaustive yet representative directory of previous studies on the subject is summarized in Table 1, providing specific information on the aforementioned five key topics for each study. A critical review on this categorized literature summary leads to the following remarks:

- A significant number of suggested algorithms are limited to R/C sections, consisting of a single concrete surface and a group of individual fibers for reinforcement bars (i.e. two distinct materials). In order to bypass such limitations, the first prerequisite of the present methodology will be the unconditional section complexity, i.e. unlimited number of section components (surfaces, fiber groups and – newly introduced – lines), each assigned to a different material constitutive law.
- Most studies impose restrictions on material stress–strain constitutive laws, usually in the form of specific Code directives (e.g. [5,6]), or more elaborate piecewise polynomial laws (e.g. [7,8]). For this reason, the second prerequisite of the present methodology will be the use of fully arbitrary material

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Table 1
Features of selected previous studies on the analysis of arbitrary R/C and composite sections.

Authors	Section type	Material constitutive law for surfaces	Section subdivision	Stress integration	Solution strategies
Charalampakis and Koumousis [8]	Composite	Piecewise up to cubic polynomials	Curvilinear trapezoids	Closed-form per trapezoid	Derivative-free (Brent)
Chen et al. [16]	Composite	Parabolic-linear	No subdivisions	Closed-form	Derivative-free (regula-falsi)
Chiorean [18]	Composite	Parabolic-linear with softening	No subdivisions	Green/Gauss-Lobatto with adaptive bisection	With derivatives (arc-length)
Rosati et al. [5]	Composite	Parabolic-linear	One polygon per $\sigma-\varepsilon$ part	Closed-form per polygon	With derivatives (NR)
Sfakianakis [15]	Composite	Parabolic-linear with softening	No subdivisions	Fiber integration	Incremental search
Sousa and Muniz [7]	Composite	Piecewise up to cubic polynomials	One polygon per $\sigma-\varepsilon$ part	Closed-form per polygon	–
Brøndum–Nielsen [25], Dundar and Sahin [22], Yen [21]	Reinforced concrete	Constant (rectangular)	No subdivisions	Closed-form	With derivatives as finite differences (NR)
Bonet et al. [1]	Reinforced concrete	Piecewise non-polynomial (arbitrary)	Polygons (thick layers) per $\sigma-\varepsilon$ part	2D Gauss or Green/Gauss per polygon	–
Fafitis [17]	Reinforced concrete	Parabolic-linear	No subdivisions	Green/Gauss	–
Pallarés et al. [6]	Reinforced concrete	Non-polynomial – linear (EC2)	No subdivisions	Closed-form	With derivatives (NR)
Penelis [3], De Vivo and Rosati [26], Alfano et al. [27]	Reinforced concrete	Parabolic-linear	One polygon per $\sigma-\varepsilon$ part	Closed-form per polygon	With derivatives (NR)
Rodriguez and Ochoa [20]	Reinforced concrete	Parabolic-linear with softening	Trapezoids	Closed-form per trapezoid	With derivatives (NR)
Werner [4]	Reinforced concrete	Parabolic-linear	One polygon per $\sigma-\varepsilon$ part	Closed-form per polygon	Nested iterations
Yau et al. [23]	Reinforced concrete	Constant (rectangular)	No subdivisions	Closed-form	Derivative-free (regula-falsi)
Suggested methodology	Composite	Piecewise non-polynomial (arbitrary)	No subdivisions	Green/Gauss with adaptive strain-mapping	Derivative-free (Brent)

constitutive laws in piecewise form, as reported only in Bonet et al. [1]. This enables the analysis of sections with materials of non-polynomial stress–strain relationships (e.g. [9] for high-strength concrete and nonlinear analysis, [10] for confined concrete and [11,12] for plain concrete).

- The most critical element that affects both the accuracy and speed of a section analysis algorithm is the stress integration scheme. There are three major paths that can be followed, namely (a) fiber integration (e.g. [13–15]), (b) analytical integration using closed-form functions (e.g. [8,16]) and (c) numerical integration, usually in a form of Gaussian sampling on a Green path integral (e.g. [17,18]). The first approach is both approximate and slow, requiring an increased fiber mesh density and a proportional number of arithmetic operations to reach an acceptable level of accuracy. Therefore, this method is now obsolete for ultimate strength section analysis and is used only for non-cylindrical stress fields, e.g. in cases of cyclic loading and/or load path dependency [1]. As far as the second approach is concerned, its main advantage is that it yields exact and quick results (especially for lower order functions), it is however totally restricted to a specific stress–strain expression, which is in conflict with the aforementioned second prerequisite. Consequently, the sole path to provide a generalized solution for arbitrary material constitutive laws is the implementation of a suitable *numerical* integration scheme. However, it has been reported that, in certain cases, numerical integration may be more expensive than analytical methods, while low order numerical integration may yield unacceptably large errors [1,19] because the required order of numerical integration is not known *a priori* [8]. These issues will be confirmed and eventually addressed in the present methodology, by applying an improved *adaptive strain-mapped* Gaussian sampling on a Green path integral, demonstrating fast execution with customizable accuracy, which expresses the third prerequisite of the present method.
- A closely related issue to the aforementioned stress integration efficiency is whether section subdivisions are required in the employed integration scheme. There are many studies (e.g. [1,7,8,20]) where this geometric manipulation is imposed prior to stress integration, naturally leading to a reduction in execution speed. For this reason, the suggested scheme will be formulated without need for any subdivisions (e.g. [6,18]).
- The majority of solution strategies presented in the literature, for applying force equilibrium conditions, are based on secant or tangent schemes (e.g. Newton-Raphson), which require the prior calculation of derivative measures (e.g. section stiffness). However, the disadvantages of (a) the additional computational cost for calculating derivatives (e.g. using finite differences [21,22]) and (b) the inherent non-convergence issues related to secant/tangent methods, has led some researchers to adopt simpler and more straightforward derivative-free procedures [8,16,23], which, under certain conditions, demonstrate stable and fast performance. Following the third prerequisite, the present methodology will apply advanced derivative-free methods [8], providing, where necessary, various techniques to guarantee convergence.

In order to satisfy the aforementioned three prerequisites of the suggested methodology, namely (a) unrestrained section complexity (b) arbitrary material constitutive laws and (c) fastest possible execution with customizable accuracy, a new feature set is suggested in the last row of Table 1. In the subsequent chapters, the mathematical formulation of the methodology will be deployed, together with validation tests, case studies, extended comparisons with previous studies and benchmarks. The ultimate goal of the present study is to suggest a new robust and fast procedure for