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Simulation of hydraulic fracturing in rock mass using a smeared crack model

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ABSTRACT

In this paper, a 3D smeared crack model with an evolutionary characteristic length is developed for the analysis of hydraulic fracturing in rock mass. A non-linear fluid–solid coupling finite element model is established and method for smeared cracking finite element model is presented. The evolutionary characteristic length method focuses on continuous adaptation of the crack band width which is based on the incremental finite element solution and the idea of nonlocal continuum. An interface coupling method is proposed to achieve the transition from a coarser mesh to a finer one. The results of two numerical examples indicate the superiority of the present formulation over the standard smeared crack model.

1. Introduction

Hydraulic fracturing is a complicated process to simulate, as it involves the coupling of four processes [1,2]: (i) the mechanical deformation induced by the fluid pressure on the fracture surfaces; (ii) the flow of fluid within the fracture; (iii) the fracture propagation; and (iv) the leak-off of fluid from the fracture into the surrounding rock. Therefore, simulation of hydraulic fracturing in rock mass, arguably one of the most challenging computational problems in geoengineering, has been the subject of numerous investigations since the pioneering work of Khristianovic and Zheltov [3].

Due to the complication of hydraulic fracturing, analytical solutions are impossible for most practical problems. Instead, numerical simulation methods are widely used, although many assumptions and simplifications are made in existing numerical studies. For example, Perkins and Kern [4] adapted the classic Sneddon plane strain crack solution to develop the so called PK model. Later, Nordgren [5] adapted the PK model to formulate the PKN model, which included the effects of fluid loss. Khristianovic and Zheltov [3], and Geertsma and de Klerk [6] independently developed the so-called KGD model. Pseudo-3D (P3D) model [7] and planar 3D (PL3D) model [8,9] are based on PKN model or KGD model, and also have a few of assumptions and simplifications. The P3D models are built on the basic assumption that the reservoir elastic properties are homogeneous, and averaged over all layers containing the fracture height. The PL3D models assume that the fracture footprint and the coupled fluid flow equation are described by a 2D mesh of cells, typically a moving triangular mesh or a fixed rectangular mesh, oriented in a (vertical) plane. There have also been attempts to simulate fully 3D hydraulic fractures with limited success [10]. The computational cost on such coupled systems is still excessive, even with today's powerful computational resources.

Many numerical methods [11–16] have been developed for the hydraulic fracture simulation. For example, Bazant and Ohtsubo [16] analyzed water circulation in hydraulic fracture by finite element method. Among these methods, the boundary integral equation or the displacement discontinuity methods cannot be efficiently used to solve the elasticity equation relating the fluid pressure to the fracture opening. The finite element method overcomes naturally this limitation at the expense that the mesh has to conform exactly to the fracture geometry at all times, implying the use of complicated remeshing algorithms in order to track crack propagation [11,12]. Lecampion [17] investigated the use of the extended finite element method (XFEM) for hydraulic fracture problems in order to avoid remeshing. However, the XFEM also needs objective crack propagation criteria, which are usually based on quantities such as crack-tip stresses and stress intensity factors (SIFs). Accurate calculation of these quantities in XFEM necessitates fine crack-tip meshes. This means that fine meshes in the whole domain of analysis are needed if cracks are unknown in priori, leading to high computational cost.

The smeared crack approach is popular for the numerical simulation of tensile failure processes in rock mass. It describes a

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cracked solid by an equivalent anisotropic continuum with degraded material properties in the direction normal to the crack orientation and no remeshing is needed. Since the advent of the smeared crack concept by Rashid [18], some improvements have been made, most notably, the crack band theory developed by Bazant and Oh [19]. One of the fundamental components in nonlinear problems undergoing localized damage is the existence of an internal length parameter. This characteristic length determines the width of the localization zone, namely, the crack band width w_c. When this parameter is related to the adopted finite element size, the spurious mesh dependency on the structural load-deformation response can be eliminated. But it requires continuous modification of the topology of the finite element mesh in order to adapt the finite element size to the present level of cracking (Upon increasing the damage level, cracking tends to localize in a band of decreasing width [20]). Bazant [21] and Oliver [22] have attempted to rationalize the arbitrariness of the choice of the crack band width. Bazant has used stability analysis to determine w_c. As he states, this stability analysis seems useful in principle, but not in practice, as it is not known how to perform this analysis. Oliver has analyzed a singular band in a 2D domain and the crack band width is related to the crack orientation and the characteristics of the finite element interpolation functions. But it renders the estimated value of the crack band width constant throughout the entire loading history. Mosalam and Paulino [20] have developed an evolutionary characteristic length method for 2D smeared crack model.

In this paper, a non-linear full fluid–solid coupling finite element model was proposed. The fluid–solid coupling theory is used to capture the behavior of rock, and the damage mechanics criterion is adopted to simulate the fracture initiation and propagation. An evolutionary characteristic length method for 3D smeared crack model is introduced and a practical adaptation of the crack band width is developed. The crack band width is treated as a parameter in the constitutive model, which greatly simplifies the technique. An interface coupling method is proposed to achieve the transition from a coarser mesh to a finer one and the principle theory of the interface coupling method is introduced. Finally, two numerical examples are presented to show the accuracy and efficiency of the characteristic length method for smeared cracking.

2. Hydraulic fracturing finite element model

2.1. Fluid–solid coupling equation

Equilibrium can be written in the form of virtual work principle for the volume under its current configuration at time t as [23]

$$\int_V (\boldsymbol{\sigma}' - p\mathbf{I}) : \delta\boldsymbol{\varepsilon} dV = \int_S \mathbf{t} \cdot \delta\mathbf{u} dS + \int_V \mathbf{f} \cdot \delta\mathbf{u} dV \tag{1}$$

where $\boldsymbol{\sigma}'$ and $\delta\boldsymbol{\varepsilon} = sym(\partial\delta\mathbf{u}/\partial\mathbf{x})$ are the effective stress and virtual rate of deformation respectively, p is the pore pressure, $\mathbf{t}$ and $\mathbf{f}$ are the surface traction per unit area and body force per unit volume respectively, $\delta\mathbf{u}$ is the virtual deformation, and $\mathbf{I}$ is unit matrix.

2.2. Continuity equation of fluid flow in porous medium

The same set of finite element mesh is used for the fluid flow analysis and the solid phase stress analysis. Equating the time rate of change of the total mass of fluid in the control volume V to the mass of fluid crossing the surface S per unit time gives the fluid mass continuity equation in the following form

$$\frac{d}{dt}\left(\int_V \rho n_p dV\right) + \int_S \rho n_p \mathbf{n} \cdot \mathbf{v}_w dS = 0 \tag{2}$$

where ρ, n_p and $\mathbf{v}_w$ are the mass density of the fluid, the porosity of the medium and the average velocity of the fluid relative to the solid phase (the seepage velocity) respectively, and $\mathbf{n}$ is the outward normal to S.

The fluid flow is described by Darcy's law as

$$\mathbf{v}_w = -\frac{1}{n_p \rho \mathbf{g}} \mathbf{k} \cdot \left(\frac{\partial p}{\partial \mathbf{x}} - \rho \mathbf{g}\right) \tag{3}$$

where $\mathbf{k}$ and $\mathbf{g} = -g\partial z/\partial\mathbf{x}$ are the permeability of the medium and the gravitational acceleration respectively.

2.3. Flow within crack elements

The fluid flow in the crack elements and fluid leakoff effect are also taken into account. The fluid flow model in the crack element enables the fluid pressure on the crack element surfaces to be incorporated in the modelling of hydraulically driven fractures. The fluid constitutive response comprises the tangential flow within the fracture and the normal flow across the fracture. The flow patterns of pore fluid in crack elements are depicted in Fig. 1.

The tangential flow in crack element is described with the following equation [23].

$$\mathbf{q}\delta_{cr} = -\frac{\delta_{cr}^3}{12\mu} \nabla p \tag{4}$$

where $\mathbf{q}$ is the volume flow rate density vector, δ_{cr} is the opening displacement of the crack surfaces, ∇p is the pressure gradient along the crack element, μ is the fluid viscosity.

The normal flow is described as

$$\begin{cases} q_t = c_t(p_i - p_t) \\ q_b = c_b(p_i - p_b) \end{cases} \tag{5}$$

where q_t and q_b are the flow rates on the top and bottom surfaces of crack element, respectively, c_t and c_b are the leakoff coefficients on the top and bottom surfaces, respectively, p_i is the internal pressure of crack element, p_t and p_b are the pore pressures on the top and bottom surfaces, respectively.

3. An evolutionary characteristic length method

The smeared crack approach is a continuum approach for fracture mechanics in which local discontinuities are distributed over a certain tributary area within the finite element. Accordingly, crack strain can be defined as a function of the relative displacement of the crack surfaces and some length parameter over which this displacement jump is distributed. The introduction of such characteristic length allows modeling of the cracked material in terms of stress–strain relations.

A family of crack normal stress vs. crack normal strain ($s_{cr} - \varepsilon_{cr}$) relations which was originally used by Reinhardt [24] for concrete is considered herein. This family can apply to rock based on

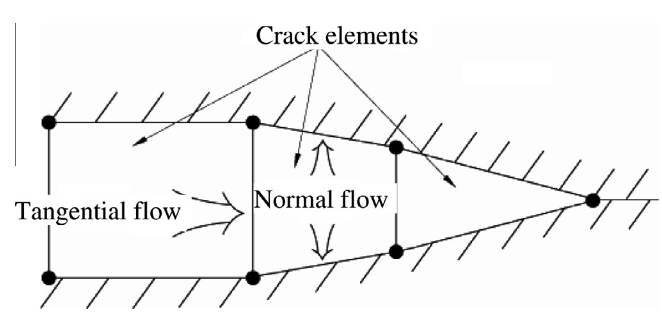

Fig. 1. Flow patterns of pore fluid in crack elements.