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Probabilistic fracture mechanics with uncertainty in crack size and orientation using the scaled boundary finite element method

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ABSTRACT

The geometry of cracks in a structure are often difficult to determine accurately, leading to uncertainties in structural analysis. This paper presents a probabilistic fracture mechanics (PFM) approach to evaluate the reliability of cracked structures considering the uncertainty in crack geometry. The shape sensitivity analysis of the stress intensity factor (SIF) is performed efficiently using the scaled boundary finite element method (SBFEM). No remeshing is required as the size and orientation of a crack vary. Reliability is estimated using various probabilistic techniques. Numerical examples demonstrate the accuracy and simplicity of the present method.

1. Introduction

It is common for ageing infrastructure such as dams, bridges, and buildings to experience cracking, leading to increased safety concerns. On the smaller scale, cracking is the dominant mode of failure in advanced composite devices such as oxide fuel cell stacks and printed electronics used in various engineering field applications today. The geometry of cracks, such as the size and orientation, can strongly affect the reliability of the structure. However, a considerable amount of uncertainties often exist in the measurement of crack geometry. In addition, there are uncertainties in qualifying the material properties and applied loads for a structural analysis. In fact, progressive deterioration of concrete and corrosion of steel usually lead to significant variations of system parameters and performance over the lifetime of a structure [1]. It is widely accepted that a probability based approach for structural design and safety evaluation is more rational than a simple deterministic approach based on the factor of safety.

Probabilistic fracture mechanics (PFM) provides a rational framework for the safety assessment of cracked structures through probabilistic analysis techniques. Some common techniques include the Monte Carlo simulation (MCS) and the first/second-order reliability methods (FORM/SORM). In all these methods, the failure probability is predicted by modelling the uncertainties in the applied load, material properties and crack geometry as random variables which are defined by specific probability distribution functions. A sensitivity analysis has to be performed to determine the sensitivity of the fracture response of the structure, such as the stress intensity factor (SIF), with respect to material properties and loading (size sensitivity) and the crack geometry (shape sensitivity). While size sensitivity can be dealt with easily by current finite element based methods, numerical and computational modelling of shape sensitivity remains a major challenge, particularly when there is a high uncertainty in the crack geometry.

The shape sensitivity analysis can yield either (1) response grid, i.e. the SIF calculated for a set of discrete crack sizes and orientations, which is inputted into the MCS; (2) derivatives of the SIF, with respect to the crack size and orientation, which are required for the FORM/SORM. Although predetermined explicit formulas of the SIF can be used (e.g. [2]), their application to fracture mechanics is limited owing to complexities in loading, material behavior and crack geometry [3]. To obtain the response grid requires repetitive calculations at discrete points of crack size and orientation. Direct application of, say, the finite element method (FEM) to model this is evidentally a brute force approach (e.g. [4–6]) and needs numerous deterministic analyses at the expense of excessive remeshing and computational effort, especially near the crack tip region. The boundary element method (BEM) offers an alternative approach, replacing the domain mesh with a simpler boundary mesh (e.g. [7–9]). However, since the crack surface forms part of the boundary it still requires discretization and remeshing. Several techniques to improve the numerical effectiveness of the FEM in determining the derivatives of the SIF have been proposed. One such example is the well recognized virtual crack extension

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(VCE) technique, originally developed by Parks [10] and Hellen [11]. Hwang et al. [12–14] and Hwang and Ingraffea [15] followed on by adopting modified formulations developed by Lin and Abel [16] to investigate first/second-order derivatives of multiple crack systems, axisymmetric stress states and crack-face and thermal loading cases. Latter methods include those based on continuum mechanics theory, namely the material derivative formulations. Unlike the VCE technique, mesh perturbation is not a fundamental requirement in order to calculate the energy release rate or the SIF. Rao and Rahman [17–19], Rahman and Rao [20] and Rahman and Chen [21] continued with this line of work to analyze shape sensitivity of cracks in isotropic and orthotropic functionally graded materials for mode-I and mixed-mode loading cases. Reddy and Rao [22–25] and Rao and Reddy [26] later introduced the well known fractal finite element method (FFEM) and extended it by adopting the material derivative concept to obtain derivatives of the SIF.

Recently, an innovative numerical tool known as the scaled boundary finite element method (SBFEM) was introduced by Song and Wolf [27] possessing promising characteristics that can greatly simplify shape sensitivity analysis. The SBFEM is a semi-analytical fundamental-solutionless method which combines the advantages of both the finite element formulations and the boundary element discretization. Unlike the BEM, no fundamental solution is required and unlike the FEM, generally, only the boundary need be meshed thereby reducing the spatial dimension of the problem by one. The stress singularity at a crack tip is expressed analytically. Special elements or numerical techniques are no longer required for fracture analysis. High accuracy and efficiency of the SBFEM in evaluating the SIF of cracks in homogeneous materials and bimaterial interfaces have been demonstrated in [28–34]. Chidgzey and Deeks [35] made the interesting connection between the coefficients in Williams power series expansion [36] of displacement and stress fields and the SBFEM solution. Not only the SIF, but also the T-stress and coefficients of higher order terms are determined directly by the SBFEM. Advancements to the application of the SBFEM to other areas have also been growing in recent times. In [37] a parallel algorithm based on the coupled FEM/SBFEM is introduced for large scaled soil-structure interaction. Birk and Behnke [38] also derived equations to further the SBFEM for analysis of 3D-layered continua such as rigid circular and square foundations embedded in or resting on the surface of layered homogeneous or inhomogeneous soil deposits over rigid bedrock. Liu and Lin [39,40] and Lin et al. [41] extended the SBFEM formulations to evaluate eigenvalues of waveguide structures including quadruple corner-cut ridged circular, rectangular, L-shaped, vaned rectangular, square and elliptical waveguides. Dynamic crack propagation problems are modelled for the first time in [42] using a flexible SBFEM remeshing algorithm which was later modified in [43] to incorporate the use of polygon scaled boundary finite elements around the crack tip to increase efficiency in the SIF calculation.

This paper presents a procedure to perform the shape sensitivity of linear elastic cracked structures efficiently using the novel scaled boundary finite element method (SBFEM), in conjunction to the MCS and the FORM/SORM for reliability estimation. It takes advantage of the fact that the SBFEM mesh is limited to the boundary, excluding the crack faces. A shape sensitivity analysis is performed by simply changing the position of the crack tip. Only one mesh is sufficient to cover a large range of variation of crack size and orientation. The whole analysis can be easily automated as no remeshing is involved. This procedure is applied to investigate the shape sensitivity and reliability of cracked plates. The crack orientation, as well as the crack size, is considered as an uncertainty variable, a case seldom reported in the literature.

2. Scaled boundary finite element method

2.1. Fracture application

To illustrate the concept of shape sensitivity using the scaled boundary finite element method (SBFEM), the arbitrary homogeneous cracked domain shown in Fig. 1 is considered. The crack size and orientation are, a, and, γ, respectively. The scaling centre, O, is selected at the crack tip. The geometry of the domain satisfies the scaling requirement, i.e. the enclosed boundary is visible from the crack tip. The boundary is discretized into elements, S^e (superscript e – element). On each element, a local coordinate, η, bounded by $-1 \leqslant \eta \leqslant +1$ is introduced. The physical domain, with coordinates $(\hat{x}, \hat{y})$, is generated by scaling the elements along the radial direction, ξ, to the scaling centre, O, forming such an area, V^e, where $0 \leqslant \xi \leqslant 1$ applies ($\xi = 0$ at scaling centre; $\xi = 1$ at boundary). The crack opening is modelled by 2 independent nodes, A and B in Fig. 1, at the boundary. The crack faces OA and OB are generated by scaling these nodes to the scaling centre and require no discretization as evident in the figure. This is one of the many distinguishing attributes of the SBFEM for fracture applications. Coincidentally, ξ and η are called the scaled boundary coordinates. To avoid the Jacobian of the scaled boundary coordinate transformation approaching zero, the acute angle, α, formed by any radial line and the boundary should not be too small. Numerical investigations within this paper show that the results are not sensitive to the angle given that it is larger than 5° and the mesh on the boundary is sufficiently fine to represent the displacement variation along the circumference.

The scaled element, S^e, is isolated in Fig. 2 with end nodes 1 ($\eta = -1$) and 2 ($\eta = +1$) indicated. Nodal displacement functions, $\{u(\xi)\}$, are introduced along the radial lines passing through the scaling centre, O, and the nodes on the boundary, while along the circumferential direction, η, the displacements can be interpolated by,

$$\{u(\xi, \eta)\} = [N(\eta)]\{u(\xi)\}, \tag{1}$$

where $[N(\eta)]$ are nodal shape functions defined in the local direction, η. Applying a weighted residual technique in the same direction (see [28]) or the virtual work principle (see [44]) then leads to the following scaled boundary finite element equation in displacement,

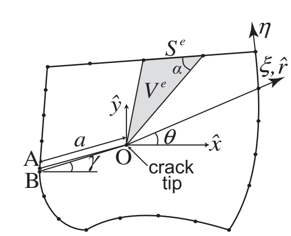

Fig. 1. Scaled boundary transformation.

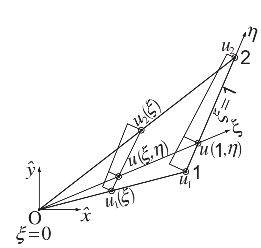

Fig. 2. Displacement of element S^e in scaled boundary coordinates.