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A consistent iterative scheme for 2D and 3D cohesive crack analysis in XFEM

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ABSTRACT

This paper presents a consistent algorithm for cohesive crack modeling with the extended finite element method (XFEM). A new formulation for the transformation from global to local quantities for the cohesive law is presented. This formulation works as a generic shell around any cohesive law so that the cohesive law needs only to be implemented in two dimensions while it can be used in both two and three dimensional analysis. Particular attention is paid to consistent linearization of the contribution from the cohesive tractions. The proposed formulation is tested on three relevant examples of three-dimensional analysis.

1. Introduction

Engineering structures have to be analyzed with respect to their failure possibilities in order to ensure their safety. Tensile stresses are particularly dangerous to many materials because they may cause decohesion of the material due to a gradual loss of microstructural load-bearing capacity. Eventually a crack appears where displacements are not continuous on the structural level of observation. The analysis of fracture problems is usually carried out with either damage mechanics or fracture mechanics. In linear elastic fracture mechanics, the failure of a material is characterized by the growth of a discrete crack through an otherwise intact medium. In most materials, however, the real crack is preceded by the so-called fracture process zone where tensile stresses decrease. The materials where the fracture process zone is not negligibly small are usually called quasi-brittle materials. An accurate description of the failure of such materials requires that the fracture process zone is taken into account in the analysis. A popular way to do this is with the so-called cohesive crack model [1–3]. In the cohesive crack model it is assumed that near the crack tip tractions act from one crack face to the other. These tractions decrease as the crack opening increases and eventually the crack becomes traction-free.

Computational modelling of the crack process was usually carried out by finite element method (FEM) e.g. [4,5]. The element–free Galerkin method (EFGM) was also used in application of crack growth analysis (e.g. [6–9]), or the method based on coupled FEM–EFGM (e.g. [10,11]). Nowadays the methods based on partition of unity method (PUM) are seen as the most accurate technology for crack propagation analysis. The extended finite element method (XFEM) is the most widely used [12–15], but extended EFGM can also applied [16,10].

The main advantage of computer methods based on PUM is the fact that arbitrary functions can be incorporated in the approximation. In the case of fracture analysis, the Heaviside step function is used to incorporate the discontinuity over the crack surface. An arbitrary number of additional functions can be used in the formulation, which is why additional enrichment functions are sometimes used near the crack tip [12,17,11,18–21]. These additional enrichment functions usually refer to an analytical solution for the displacement field around the crack tip. In the cohesive crack model there is no singularity at the crack tip and therefore the crack tip enrichment with singular functions is not necessary. With cohesive methods, a stress criterion is used for crack initiation, while the fracture toughness plays a role in the propagation through the cohesive law. This combined approach with fracture stress and toughness as material parameters is typical for quasi-brittle materials.

In the cohesive crack model, cohesive forces act along the crack in a fracture process zone. At every point on the crack surface, there is a crack opening vector that can be decomposed into the normal w_n and sliding w_s parts, where normal and sliding directions refer to the crack orientation. In most analyses, the cohesive forces act in the normal direction and are described by a constitutive cohesion law in terms w_n. When mixed-mode cracking is analyzed, additional forces that resist the sliding deformation

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between the crack faces can come into play. Then, an additional constitutive law is set for these sliding forces which depend on both crack opening parts w_n and w_s as in [13]. Sometimes the total crack opening is used to construct the cohesion traction-separation law (e.g. [8,18,22]). In this paper, the general assumptions are made that normal and sliding cohesion forces are taken into account and that both depend on w_n and w_s. Based on these assumptions general relations are derived.

The cohesive crack propagation analysis is a nonlinear problem. Firstly, the nonlinearity of the process is connected with the propagation of the discontinuity. It results in a decrease in the stiffness of the structure. Secondly, the cohesion forces along the crack depend on the actual solution. In order to find the solution of this nonlinear problem, a Newton–Raphson scheme must be applied, which means that the state in which internal forces are in equilibrium with external forces is found iteratively. This iterative procedure is most effective when a tangential stiffness matrix is calculated. Therefore, a procedure is needed to linearize the relation between cohesive forces and the crack opening vector.

The tangential stiffness matrix includes a second order tangent cohesive stiffness matrix $\mathbf{C}$ from the cohesion traction-separation law and the shape functions jump. This kind of matrix was used for example in [13–15,23] but it was evaluated for standard XFEM formulation with the Heaviside step function and basic cohesion constitutive law. In [18] the tangent matrix was evaluated for local coordinate system connected with the crack line. A more general idea was proposed in [11,16] where the matrix was developed by differentiating the cohesion force vector with respect to displacement jump vector in the global coordinate system which was applied to mode I cohesive cracks. In this paper the relations from [11] are generalized to cracks in three dimensions (3D) with general cohesive constitutive laws. The tangent matrix is evaluated in global coordinate system, while the cohesive law is evaluated in a two-dimensional (2D) local coordinate frame aligned with the crack surface.

In the past decade there are plenty of works concerning fracture simulation by XFEM. Some example analysis of three-dimensional crack analysis can be found in [24–26] while recent developments connected with cohesive crack models have been presented in [27–33]. A review of XFEM has been given in [9].

The paper is organized as follows: In Section 2 a mathematical model for the quasi-static cohesive cracking problem is developed. In Section 3 a new approach is presented for calculating and applying cohesion forces in cracks in 2D and 3D. The presented algorithm is verified by examples presented in Section 4. In Section 5 conclusions are presented.

2. Mathematical model

The computational model in the paper is derived for full 3D case. However, the relations that are derived below can however be simplified to the 2D case in a plane stress or plane strain state. It is assumed that:

1. strains and displacement are small,
2. the relation between stress and strain is linear,
3. there is a crack in the body denoted by S_d. In 3D the crack is a curved plane, in 2D the crack is a curved line,
4. along S_d the displacement field is discontinuous,
5. there is a fracture process zone in the crack that is modeled by cohesion forces $\mathbf{t}_c$.

In the paper Voigt notation is used for stress $\boldsymbol{\sigma}$, strain $\boldsymbol{\varepsilon}$ and matrix $\mathbf{m}$ representing the normal to the surface of the analyzed body.

The analysis begins with the standard equilibrium equation (momentum balance) that is valid at each point of the considered isotropic solid and for each moment of time:

$$\mathbf{L}^{\mathrm{T}}\boldsymbol{\sigma} + \mathbf{b} = \mathbf{0} \quad \text{in } V, \tag{1}$$

where $\mathbf{b}$ is the body force vector and $\mathbf{L}$ a matrix of differential operators.

The equation is completed by the standard boundary conditions and equilibrium conditions for the crack plane:

$$\begin{aligned} &\mathbf{m}^{\mathrm{T}}\boldsymbol{\sigma} = \mathbf{t} \quad \text{on } S_\sigma, \\ &\mathbf{u} = \hat{\mathbf{u}} \quad \text{on } S_u, \\ &\mathbf{m}_d^{\mathrm{T}}\boldsymbol{\sigma} = \mathbf{t}_c \quad \text{on } S_d, \end{aligned} \tag{2}$$

where $\mathbf{t}$ is the vector of external tractions, $\mathbf{u}$ is the displacement vector, $\hat{\mathbf{u}}$ is the vector of prescribed displacements, $\mathbf{m}_d$ represents the normal to the crack surface in Voigt notation, and $\mathbf{t}_c$ is the vector of cohesive forces along the crack surface.

The local mathematical model from Eqs. (1) and (2) is reworked into a global weak formulation using the weighted residual approach, where a test function $\mathbf{v}_{\mathrm{u}}$ is used. The test function $\mathbf{v}_{\mathrm{u}}$ and displacement field $\mathbf{u}$ are discontinuous in S_d. Therefore Dirac's delta appears in S_d while differentiating the fields

$$\mathbf{Lu} = \begin{cases} \mathbf{Lu} & \text{if} \quad \mathbf{x} \notin S_d, \\ \delta_{S_d}\mathbf{m}_d[\![\mathbf{u}]\!] & \text{if} \quad \mathbf{x} \in S_d. \end{cases} \tag{3}$$

where:

δ_{S_d} – Dirac's delta along crack S_d,
$\mathbf{m}_d$ – normal to S_d in Voigt notation,
$[\![\cdot]\!]$ – function discontinuity operator.

Supposing that $\mathbf{n}_d$ is a normal vector to S_d in $\mathbf{x} \in S_d$, the discontinuity of the test function (or any other quantity) is

$$[\![\mathbf{v}_{\mathrm{u}}]\!](\mathbf{x}) = \lim_{\lambda\to 0}\mathbf{v}_{\mathrm{u}}(\mathbf{x}+\lambda\mathbf{n}_d) - \lim_{\lambda\to 0}\mathbf{v}_{\mathrm{u}}(\mathbf{x}-\lambda\mathbf{n}_d) = \mathbf{v}_{\mathrm{u}}^{+}(\mathbf{x}) - \mathbf{v}_{\mathrm{u}}^{-}(\mathbf{x}). \tag{4}$$

In that case the equilibrium equation in weak form at time $t+\Delta t$ is

$$\begin{aligned} &\int_V (\mathbf{L}\mathbf{v}_{\mathrm{u}})^{\mathrm{T}}\boldsymbol{\sigma}^{t+\Delta t}\,\mathrm{d}V + \int_V \delta_{S_d}[\![\mathbf{v}_{\mathrm{u}}]\!]^{\mathrm{T}}\mathbf{m}_d^{\mathrm{T}}\boldsymbol{\sigma}^{t+\Delta t}\,\mathrm{d}V - \int_V \mathbf{v}_{\mathrm{u}}^{\mathrm{T}}b^{t+\Delta t}\,\mathrm{d}V \\ &- \int_{S_\sigma}\mathbf{v}_{\mathrm{u}}^{\mathrm{T}}\mathbf{t}^{t+\Delta t}\,\mathrm{d}S = 0, \end{aligned} \tag{5}$$

where t is the pseudo-time connected with the external forces that is usually used in quasi-static analysis, δ_{S_d} is Dirac's delta along crack S_d.

With the definition of Dirac's delta, the weak formulation can be rewritten as:

$$\int_V (\mathbf{L}\mathbf{v}_{\mathrm{u}})^{\mathrm{T}}\boldsymbol{\sigma}^{t+\Delta t}\,\mathrm{d}V + \int_{S_d}[\![\mathbf{v}_{\mathrm{u}}]\!]^{\mathrm{T}}\mathbf{t}_c^{t+\Delta t}\,\mathrm{d}S - \int_V \mathbf{v}_{\mathrm{u}}^{\mathrm{T}}\mathbf{b}^{t+\Delta t}\,\mathrm{d}V - \int_{S_\sigma}\mathbf{v}_{\mathrm{u}}^{\mathrm{T}}\mathbf{t}^{t+\Delta t}\,\mathrm{d}S = 0. \tag{6}$$

where $\mathbf{t}_c$ depends on the discontinuity in the displacements, $[\![\mathbf{u}]\!]$, that is referred to in this paper as the crack opening vector.

For the solution of the nonlinear system of equations the standard incremental-iterative procedure is applied. It is supposed that all quantities are known for time t. The displacement in time $t+\Delta t$ is then defined as the sum of the displacement at time t and an unknown increment

$$\mathbf{u}^{t+\Delta t} = \mathbf{u}^{t} + \Delta\mathbf{u}. \tag{7}$$

The displacement increment $\Delta\mathbf{u}$ is computed iteratively and it can be defined as the limit of a sequence, where an element of the sequence is then calculated by adding a corrective increment $d\mathbf{u}$ to the previous element of the sequence:

$$\Delta\mathbf{u} = \lim_{i\to\infty}\Delta\mathbf{u}_i, \quad \Delta\mathbf{u}_{i+1} = \Delta\mathbf{u}_i + d\mathbf{u}. \tag{8}$$