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Global buckling of composite plates containing rectangular delaminations using exact stiffness analysis and smearing method

Mahdi Damghani, David Kennedy *, Carol Featherston

Cardiff School of Engineering, Queen's Buildings, The Parade, Cardiff CF24 3AA, United Kingdom

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ABSTRACT

The global buckling of a composite plate with a single rectangular delamination is studied using a smearing method and employing exact stiffness analysis and the Wittrick–Williams algorithm. Computational efficiency is achieved by avoiding discretisation into elements and non-linear analyses, making the method suitable for parametric studies in preliminary aircraft design. Numerical results for longitudinal compressive loading show the level of reduction in buckling load with increasing length and width of delamination. Global buckling strength is increased as the delamination is moved towards the plate surface, but is relatively insensitive to its widthwise and lengthwise location. The results are validated by finite element analysis.

1. Introduction

The benefits and advantages of using lightweight structures in industries such as aerospace and the automotive sector have directed engineers to the use of new materials. Composite materials are an example of the many such new man-made materials which can be tailored for specific applications. With the use of composite materials, however, certain new material imperfections can be encountered. One of these imperfections is delamination, which may occur as the result of low-velocity impact (e.g. tool drop), fatigue loading, air entrapments caused by manufacturing processes or stress concentrations at free edges. The presence of delaminations can bring about local buckling which may also affect global buckling behaviour and result in overall degradation of stiffness by a level which depends on the size, shape, and the in-plane and through-thickness positions of the delamination.

In recent years many studies have been carried out to investigate the effects of delaminations on the buckling behaviour of composite structures. Karihaloo and Stang [1] examined the pre- and post-buckling response of a strip delamination in a composite laminate analytically and experimentally. They also developed guidelines for assessing whether or not it poses a threat to the safe operation of the laminate. Lee and Park [2] studied the interaction between local and global buckling behaviours of composite laminates. They investigated the effect of various parameters, such as delamination size, aspect ratio, width-to-thickness ratio and stacking sequence on through-the-width delaminations and also the effects of location of delamination and the existence of multiple delaminations on embedded rectangular delaminations. Riccio and Gigliotti [3] presented a fast numerical method for simulation of delamination growth in delaminated composite panels using four linear analyses. The work was validated against two finite element models, with through-the-width and embedded delaminations, respectively. The numerical results obtained were compared to two- and three-dimensional numerical results. Butler et al. [4] presented a new model which could predict the compressive fatigue limit strain of composites containing barely visible impact damage (BVID). The method was based on a combination of 2D and 1D models and represented the complexity of the morphology and progression of damage during static growth of a single delamination at a critical depth within the sample. The results obtained using this method were compared with two sets of experimental results, involving the use of different materials, different stacking sequences and different levels of impact energy. They also presented an enhanced version of the model for predicting the magnitude of fatigue strain required to propagate an area of BVID at a critical delamination level [5]. The new enhanced model uses an updated propagation approach based on plate bending energy together with damage principles. Pekbey and Sayman [6] conducted experimental measurements and determined numerical solutions for the buckling of glass-fibre rectangular plates containing a single delamination. In addition, the effects of variation in structural configuration, such as ply stacking sequence, the width of the delamination and specimen geometry (width to unsupported length), were considered. In all cases, the delamination was centrally placed

* Corresponding author. Tel.: +44 29 2087 5340; fax: +44 29 2087 4939.
E-mail address: kennedyd@cf.ac.uk (D. Kennedy).

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through-the-thickness of the laminate. Compression tests were carried out on EP GC 203 glass/epoxy woven composites with a single embedded delamination built in, in order to evaluate critical buckling load. Finite element modelling was used to gain further understanding of the critical buckling load.

Much of the research carried out within the scope of stability theory relies on finite element analysis (FEA) to model the critical buckling behaviour of composite laminates with one or more given delaminated regions. This is a versatile approach, capable of handling many combinations of loading and boundary conditions for a range of delamination shapes. However it does not provide explicit or closed-form solutions, and even with today's computer hardware it can be computationally expensive. Recently Ovesy and Kharazi [7] studied the interaction between local and global buckling using a series-based analytical model which included contact phenomena. The accuracy of this approach was enhanced by the use of first order, and subsequently higher order [8], shear deformation theory.

The approach adopted in the current work employs stiffness matrices which can be considered exact, in the sense that they are derived from analytical solutions of the governing differential equations for rectangular plates whose geometry and loading are uniform in the longitudinal direction. Embedded rectangular delaminations are modelled by means of a new smearing method, in which each longitudinal strip containing such a delamination is represented by a number of constituent prismatic regions whose combined stiffness is equal to that of the actual non-prismatic strip. The effects of size, depth and lengthwise and widthwise position of a single rectangular delamination in a clamped-free composite plate under compressive loading are investigated, using an implementation of the proposed smearing method in the computer program VICONOPT [9]. The results of the analysis are compared with FEA and the computational efficiency of the present method is confirmed.

2. Exact stiffness analysis

Many authors have presented methods for determining the critical buckling loads of prismatic structures which are assembled by connecting plates together rigidly along their longitudinal edges. The use of exact stiffnesses derived from analytical solutions of the member stiffness equations for individual plates of the assembly avoids the discretisation approximations inherent in the usual finite element and finite strip methods, but results in transcendental (rather than linear) eigenproblems.

Wittrick and Williams [10] devised an algorithm which guarantees convergence to any required accuracy on the eigenvalues (i.e. critical buckling loads or natural frequencies) of a structure when such stiffnesses are used. This algorithm is used in the computer program VIPASA [11] in which the mode of buckling or vibration is assumed to vary sinusoidally in the longitudinal direction with half-wavelength λ. Eigenvalues and modes can be obtained for any half-wavelength λ specified by the user. When all plates of the assembly are isotropic or orthotropic and carry no shear load, the nodal lines are straight and perpendicular to the longitudinal (x) direction. In these cases VIPASA gives exact solutions for plate assemblies with simply supported ends, so long as λ divides exactly into the length l. When anisotropy or shear load are present the nodal lines are skewed, and hence there are spatial phase differences across the width of the plates, which are realised by using complex arithmetic. The solutions therefore only approximate simply supported ends, being quite accurate for short wavelength buckling, i.e. when $\lambda \ll l$, but becoming substantial underestimates as λ approaches l, i.e. they are very conservative for overall modes [12].

This limitation is overcome in the computer program VICON [13] by coupling together the VIPASA stiffness matrices for different half-wavelengths λ using the method of Lagrangian multipliers. VICON retains the guarantee of convergence on all required eigenvalues [14], and uses constraints to represent arbitrarily located point supports, or point connections of the plate assembly to simple elastic supporting structures consisting of transverse beam-columns. This analysis has been extended to allow point connections between two or more plate assemblies, e.g. to model riveted connections [15]. All such constraints are assumed to repeat at intervals of l to give an infinitely long plate assembly for which the buckling mode repeats over some multiple of l. The infinitely long model thus represents continuity with adjacent parts of the structure and also gives approximate solutions for a plate assembly of finite length l.

VICONOPT [9,16] incorporates both VIPASA and VICON forms of analysis, and also has postbuckling and optimisation capabilities. While most conventional structural analysis and design procedures are based on FEA, specialist software such as VICONOPT provides faster solutions of sufficient accuracy for the preliminary design of aerospace structures, e.g. for parametric studies which investigate large numbers of alternative configurations before more detailed analysis is performed on the most promising ones.

3. Problem definition and theory

3.1. Physical basis

The aim of this study is to represent a non-prismatic (delaminated) structure by a prismatic structure in which the constituent members have equal length l and each has uniform material properties along its longitudinal (x) direction, such that the overall stiffness of the prismatic structure is equal to that of the original non-prismatic structure. This representation enables embedded delaminations to be modelled using VICONOPT for the first time in order to study the buckling behaviour of such a structure, and hence to extend the capabilities of the software in preliminary aircraft design. It should be mentioned that due to the inherent assumptions in VICONOPT, each of the constituent members of the plate assembly is modelled as infinitely long and the buckling mode is assumed to repeat over a length L, which is a multiple of l (see Section 3.2). Therefore there is always some degree of approximation when modelling a structure of finite length, unless the ends are fully clamped at $x = 0$ and $x = l$.

3.2. Theory

The present study examines a rectangular composite laminated plate containing an embedded rectangular delamination. The longitudinal portion of the plate containing the delamination (which will be termed the delaminated strip) is divided into four different regions, as shown in Figs. 1 and 2. Regions 1 and 2 are the top and bottom sub-laminates having length d (=μl), width b, thicknesses h_1 and h_2 (=$h - h_1$), and stiffness matrices $\mathbf{K}_1$ and $\mathbf{K}_2$ per unit length, respectively. Regions 3a and 3b are undelaminated and each has the stiffness matrix $\mathbf{K}$ per unit length.

Including sub-laminates in a VICONOPT model is only accurate and computationally efficient if they are prismatic and have the same length (l) as the rest of the structure. Hence, the proposed methods are based on manipulating the properties of the sub-laminates so that they have length l instead of d, width b, and thicknesses h_1 and h_2, while maintaining their original stiffnesses, i.e. $\mathbf{K}_1$ and $\mathbf{K}_2$.

In VICON analysis [14], the nodal deflections $\mathbf{D}_A$ of an infinitely long plate assembly are modelled as a series

$$\mathbf{D}_A = \sum_{m=-\infty}^{\infty} \mathbf{D}_m \exp\left(\frac{i\pi x}{\lambda_m}\right) \tag{1}$$