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Coupled plate energy models at mid- and high-frequency vibrations

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ABSTRACT

At mid- and high-frequency bands, displacement-based approaches such as the finite element method (FEM) create too large models, while energy-based methods, such as statistical energy analysis, produce smaller ones, but without spatial variation. Energy flow analysis (EFA) can produce compact models that include spatial variation; however, their analytical solution makes them difficult to handle for built-up structures. To overcome this issue, the energy finite element method (EFEM), a finite element solution of EFA, was proposed. A more accurate alternative to EFEM is the energy spectral element method (ESEM). It is a matrix methodology applied to EFA similar in style to FEM, with one significant difference being the use of the analytical solution as interpolation functions. Simulated results obtained by EFEM and ESEM are analysed and compared with each other and with the spectral element method, which is used as a reference.

1. Introduction

Several vibroacoustic prediction tools have been developed for mid- and high-frequency ranges; a commonly used technique is statistical energy analysis (SEA) [1]. However, a primary drawback to SEA is to provide only one energy level for each subsystem. It is generally accepted that energy flow analysis (EFA) improves SEA, since it allows the spatial variation of energy within each subsystem. Originally proposed by Wholever and Bernhard [2] for structures like rod and beam, EFA's formulation is based on the analogy between mechanical and thermal energy flow. It provides an approximate analytical energy solution with far fewer parameters and less computational effort than the exact analytical energy solution using displacement formulations. Studies on EFA have been extended to flexural waves in thin plates, membranes, and vibroacoustic problems [3–5].

As an alternative to the analytical solution, the EFA differential equations can be solved with standard finite element approximations; this approach is called the energy finite element method (EFEM). EFEM has been applied to structures like rod, beam, membrane, plate, and acoustic cavity [3,5,6].

The spectral element method is a technique relatively new which is similar to FEM in some aspects, mainly because of the matrix methodology employed. The difference between them is on the numbers of elements used to analyse a structural problem. In the SEM the number of elements used only need to coincide with the number of discontinuities present at the structure to be analysed. It can be shown that one spectral element is equivalent to an infinite number of finite elements [7].

Proposed originally by Santos et al. [9] for structures like rod and beam, the energy spectral element method (ESEM) consists basically into apply the same FEM matrix methodology to the energy flow analysis, but using the analytical solution as form functions. In this paper, an application of ESEM to flexural wave propagation in thin plates is presented. The method has the advantage of modelling only the energy variation, which is smoother than the displacement variation and operates efficiently over a large frequency range. ESEM is also able to maintain the accuracy of the energy density over the entire member space domain, provided there are sufficient modes in the frequency band of interest [10]. For the plate-type spectral element, the accuracy and sensitivity of the dynamic response can be monitored by the average frequency and the superposition of harmonics. The method applies to the mid- and high-frequency range, where displacement approaches are expensive and the SEA solution does not provide sufficient details for the spatial behaviour. However, there are still some issues in the use of ESEM for modelling plate elements with non-uniform geometry and the application of arbitrary boundary conditions. Furthermore, the solution of ESEM is an approximation in comparison with SEM, due to using the energy differential approximate equation on the matrix methodology of SEM.

In this work, the accuracy of ESEM and EFEM simulated results are verified by comparing them with the results from the spectral element method (SEM), developed by Doyle [7]. SEM is the exact

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analytical solution of the wave equation in the frequency domain, and uses a displacement formulation tailored with the matrix approach from the finite element method. SEM is appropriate as reference method, since the wave propagation in its members is accurately expressed. Simulated results obtained by ESEM and EFEM are also compared in order to evaluate the performance between these two different energy methods.

This paper is an updated and revised version of an earlier conference paper [8], and includes a more detailed and comprehensive formulation for the three methods (EFEM, ESEM, and SEM). An additional simulated example is presented consisting of two thin plates with a coplanar connection; where the discontinuity is not based on different plate thicknesses but on different plate material properties. Results for the energy density and energy flow calculated by all methods are compared and discussed.

2. Energy governing equations for thin plates

This section presents the fundamental theory for the energy density and energy flow (intensity) distributions in uniform thin plates, as used in energy-based and displacement-based methods.

2.1. Energy flow analyses

First, energy-based methods applied to transversally vibrating finite plates are briefly reviewed [3]. The classical governing displacement equation for a thin plate with hysteretic structural internal damping can be written as [7]:

$$D_c\nabla^4 w(x,y,t) - \rho h\frac{d^2 w(x,y,t)}{dt^2} = p(x,y,t), \tag{1}$$

where w is the out-of-plane plate displacement along the z-direction, ρ is the mass density, h is the plate thickness, and p is the excitation force. The complex flexural rigidity is given by

$$D_c = \frac{E_c h^3}{12(1-\nu^2)}, \tag{2}$$

where ν is Poisson's ratio, $E_c = E(1+i\eta)$ is a complex Young's modulus with a structural internal damping loss factor η, and $i=\sqrt{-1}$. Although solutions to Eq. (1) are available in the literature for different boundary conditions, a complete and general solution in closed form is not known. By applying the Fourier transform to both sides of Eq. (1), its spectral representation can be written as

$$D_c\nabla^4\hat{w}(x,y) - \rho h\omega^2\hat{w}(x,y) = \hat{p}(x,y), \tag{3}$$

where $\wedge$ denotes that the function has been Fourier transformed. The homogeneous form of Eq. (3) can be factored as

$$\left(\nabla^2 + k_c^2\right)\left(\nabla^2 - k_c^2\right)\hat{w}(x,y) = 0, \tag{4}$$

where $k_c^2 = (\omega^2\rho h/D_c)^{1/2}$ is the complex wave number. To analyse the energy flow in plates at mid- and high-frequencies, the far-field assumption [13] can be applied to the harmonic solution of the homogeneous form of Eq. (3), to obtain:

$$\hat{w}(x,y) = \left(A_x e^{-ik_x x} + B_x e^{ik_x x}\right)\left(A_y e^{-ik_y y} + B_y e^{ik_y y}\right), \tag{5}$$

where A_x, B_x, A_y, and B_y are arbitrary constants. For a small loss factor ($\eta \ll 1$), the wave number components of k_c are $k_x = k_{x1}(1 - i\eta/4)$ and $k_y = k_{y1}(1 - i\eta/4)$, where $k_{x1}^2 + k_{y1}^2 = k_c^2$ must be satisfied. Since the far-field solution only satisfies the factored left hand side of Eq. (4), it is an incomplete displacement solution of Eq. (3). For harmonic excitation, the time-averaged energy density for a flexural wave in a thin plate can be written as a sum of the potential and kinetic energy densities [3],

$$\langle e(x,y)\rangle = \frac{1}{4}D\left[\frac{\partial^2\hat{w}}{\partial x^2}\frac{\partial^2\hat{w}^*}{\partial x^2} + \frac{\partial^2\hat{w}}{\partial y^2}\frac{\partial^2\hat{w}^*}{\partial y^2} + 2\nu\frac{\partial^2\hat{w}}{\partial x^2}\frac{\partial^2\hat{w}^*}{\partial y^2} + 2\nu(1-\nu)\frac{\partial^2\hat{w}}{\partial x\partial y}\frac{\partial^2\hat{w}^*}{\partial x\partial y}\right] + \frac{1}{4}\rho h\omega^2\hat{w}\hat{w}^*, \tag{6}$$

where * represents the complex conjugate and D is the real part of a complex flexural rigidity. The corresponding time-averaged energy flow in the x- and y-directions can be written, respectively, as:

$$\langle q_x\rangle = \frac{1}{2}\Re\left\{i\omega D\left[\left(\frac{\partial^3\hat{w}}{\partial x^3} + \nu\frac{\partial^3\hat{w}}{\partial x\partial y^2}\right)\hat{w}^* - \left(\frac{\partial^2\hat{w}}{\partial x^2} + \nu\frac{\partial^2\hat{w}}{\partial y^2}\right)\frac{\partial\hat{w}^*}{\partial x} - (1-\nu)\frac{\partial^2\hat{w}}{\partial x\partial y}\frac{\partial\hat{w}^*}{\partial y}\right]\right\},$$
$$\langle q_y\rangle = \frac{1}{2}\Re\left\{i\omega D\left[\left(\frac{\partial^3\hat{w}}{\partial y^3} + \nu\frac{\partial^3\hat{w}}{\partial x^2\partial y}\right)\hat{w}^* - \left(\frac{\partial^2\hat{w}}{\partial y^2} + \nu\frac{\partial^2\hat{w}}{\partial x^2}\right)\frac{\partial\hat{w}^*}{\partial y} - (1-\nu)\frac{\partial^2\hat{w}}{\partial x\partial y}\frac{\partial\hat{w}^*}{\partial x}\right]\right\}, \tag{7}$$

where $\Re$ is the real part of a complex number. The far-field energy density and energy flow can be obtained by substituting Eq. (5) into Eqs. (6) and (7), but their expanded expressions display no apparent relationship. Nevertheless, reduced expressions can be obtained by neglecting all terms of η^2 and higher, as well as terms containing sinusoidal functions of the wave number. These last simplifications are equivalent to spatial averaging. For conciseness, these expanded and reduced expressions are not shown here, but they are found in [3]. The reduced expressions reveal that the energy density and energy flow are related as:

$$\langle\bar{q}(x,y)\rangle = -\frac{c_g^2}{\eta\omega}\nabla\langle\bar{e}(x,y)\rangle, \tag{8}$$

where $^-$ denotes a space-average, and $c_g = 2[\omega^2(D/\rho h)]^{1/4}$ is the group speed. From the energy conservation principle the steady-state structural energy distribution can be stated as:

$$\nabla q(x,y) = -\Pi_{diss}, \tag{9}$$

where Π_{diss} is the power dissipated in the medium. The power can be considered proportional to the energy density [1]:

$$\Pi_{diss} = \eta\omega e(x,y). \tag{10}$$

From Eqs. (8)–(10), and considering an external input power $\Pi(x,y)$, the energy equation for the far-field space and time averaged energy density of a thin plate, can be written as:

$$\nabla^2\frac{c_g^2}{\eta\omega}\langle\bar{e}(x,y)\rangle - \eta\omega\langle\bar{e}(x,y)\rangle = \Pi(x,y). \tag{11}$$

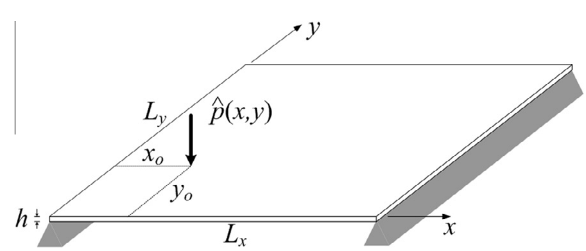

Fig. 1. Levy-type rectangular flat plate spectral element.

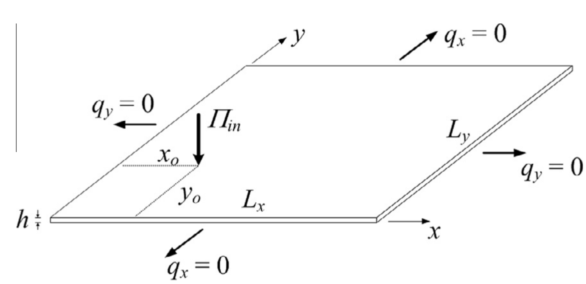

Fig. 2. Levy-type rectangular flat plate energy spectral element.