Computers and Structures 134 (2014) 112–127

Contents lists available at ScienceDirect

Computers and Structures

journal homepage: www.elsevier.com/locate/compstruc

Corotational formulation for nonlinear dynamics of beams with arbitrary thin-walled open cross-sections

Thanh-Nam Le [a,b], Jean-Marc Battini [b,*], Mohammed Hjiaj [a]

[a] Université Européenne de Bretagne, INSA de Rennes – LGCGM/Structural Engineering Research Group, 20 avenue des Buttes de Coësmes, CS 70839, 35708 Rennes Cedex 7, France
[b] KTH, Royal Institute of Technology – Department of Civil and Architectural Engineering, SE-10044 Stockholm, Sweden

ARTICLE INFO

Article history:
Received 5 July 2013
Accepted 18 November 2013
Available online 10 January 2014

Keywords:
Thin-walled
Corotational method
Nonlinear dynamics
3D beam elements
Finite rotations

ABSTRACT

A new consistent corotational formulation for nonlinear dynamics of beams with arbitrary thin-walled cross-section is presented. The novelty is that the warping deformations and the eccentricity of the shear center are fully taken into account. Therefore, additional terms are introduced in the expressions of the inertia force vector and the tangent dynamic matrix. Their contribution is then investigated considering several numerical examples. Besides, the element has seven degrees of freedom at each node and cubic shape functions are used to interpolate local transverse displacements and axial rotations. The formulation's accuracy is assessed considering five examples with comparisons against 3D-solid solutions.

1. Introduction

Dynamic analysis of flexible structures undergoing large displacements and finite rotations is an attractive research topic and a large number of finite beam formulations have been proposed in the literature. Many of them have been formulated in the total Lagrangian context [3,6,7,14,21,23,25,26,32,38,31,40,41], and in the corotational context [5,8–10,12,13,16–18,24,28,30,33,35,39]. However, the number of formulations which deal with nonlinear dynamics of beams with arbitrary thin-walled cross-sections is very limited [18].

The aim of this paper is to propose a new dynamic formulation for nonlinear analysis of beams with arbitrary thin-walled open cross-sections. The formulation is based on the corotational method. This method is a well-known approach to develop efficient beam elements for the nonlinear analysis of flexible structures. In fact, several versions of the corotational method have been proposed in the literature. The one used in this work is based on the work of Nour-Omid and Rankin [34,36], further developed by Battini and Pacoste [4] for the static analysis of beams with arbitrary cross-section. The main idea of the method is to decompose the motion of the element into rigid body and pure deformational parts. During the rigid body motion, a local coordinates system, attached to the element, moves and rotates with it. The deformational part is measured in this local system. The main interest of the approach is that different assumptions can be made to represent the local deformations.

Using this corotational framework, a dynamic formulation has been proposed by the authors for the case of 2D beams [28] and the case of 3D beams with solid and double symmetric cross-sections (Saint–Venant torsion) [30]. In these works, the same kinematic assumptions have been adopted to develop the static and the dynamic terms. In particular it has been shown that the possibility offered by the corotational method to use cubic shape functions for the local transverse displacements leads to a more efficient two-noded beam element.

The purpose of the present paper is to further develop the dynamic formulation proposed in [30] so that beams with arbitrary thin-walled open cross-sections can be studied. For that, two main ideas are used. Firstly, a seventh degree of freedom is added at each node to describe the warping of the cross-section. Consequently, the linear interpolation for the local axial rotation used in [30] is replaced by a cubic interpolation. Secondly, the main difficulty in the nonlinear analysis of beams with arbitrary cross-sections is that, due to the eccentricity of the shear center, the cross-section rotations are usually not defined at the same point. To avoid this difficulty, the kinematic description proposed by Gruttmann et al. [1] is adopted. In this approach, the warping function is modified and all the cross-section rotations are defined at the centroid.

Consequently, the inertia terms are consistent with the static ones developed by Battini and Pacoste in [4] since the same corotational kinematic description is used to derive all the terms. Regarding the dynamic terms, i.e., the inertia force vector and tangent dynamic matrix, the formulation proposed in [30] is extended

* Corresponding author.
E-mail addresses: Thanh-Nam.Le@insa-rennes.fr (T.-N. Le), jean-marc.battini@byv.kth.se (J.-M. Battini), mohammed.hjiaj@insa-rennes.fr (M. Hjiaj).

0045-7949/$ - see front matter © 2013 Elsevier Ltd. All rights reserved.
http://dx.doi.org/10.1016/j.compstruc.2013.11.005

and several additional terms are introduced. The contribution of these terms in the performance of the formulation is then investigated considering several numerical examples. Regarding the static deformational terms, i.e., the internal force vector and tangent stiffness matrix, the corotational beam element developed by Battini and Pacoste [4] is adopted. However, in order to introduce the bending shear deformations, the cubic Hermitian functions are modified as suggested in the Interdependent Interpolation Element (IIE) formulation [37].

The outline of the paper is as follows. Section 2 presents some aspects of the parametrization of finite rotations. In Section 3, the expression of the kinetic energy is derived. Section 4 is devoted to the corotational beam kinematics. Section 5 presents the derivation of the inertia force vector and the dynamic tangent matrix. The internal force vector and the tangent stiffness matrix are shortly presented in Section 6. In Section 7, five numerical examples are analyzed in order to assess the performance of the present dynamic formulation. Finally, conclusions are given in Section 8.

2. Parametrization of finite rotation

In this section, the basic relations concerning the parameterizations of finite rotations are briefly presented. For a more complete description, the reader is referred to textbooks and review papers such as [2,11,15,19,20,27].

The coordinate of a vector $\mathbf{x}_o$ that is rotated into the position $\mathbf{x}$ (see Fig. 1) is given by the relation

$$\mathbf{x} = \mathbf{R}\,\mathbf{x}_o. \tag{1}$$

Due to its orthonormality, the rotation matrix $\mathbf{R}$ can be parameterized using only three independent parameters. One possibility is to use the rotational vector defined by

$$\boldsymbol{\theta} = \theta\,\mathbf{n}, \tag{2}$$

where $\mathbf{n}$ is a unit vector defining the axis of the rotation and $\theta = (\boldsymbol{\theta}^{\mathrm{T}}\boldsymbol{\theta})^{1/2}$ is the angle of the rotation.

The relation between the rotation matrix and the rotational vector is given by the Rodrigues' formula

$$\mathbf{R} = \mathbf{I} + \frac{\sin\theta}{\theta}\widetilde{\boldsymbol{\theta}} + \frac{1-\cos\theta}{\theta^2}\widetilde{\boldsymbol{\theta}}\,\widetilde{\boldsymbol{\theta}} = \exp(\widetilde{\boldsymbol{\theta}}), \tag{3}$$

where $\widetilde{\boldsymbol{\theta}}$ is the skew matrix associated with the vector $\boldsymbol{\theta}$.

The variation of the rotation matrix in spatial and material form is given by

$$\delta\mathbf{R} = \widetilde{\delta\mathbf{w}}\mathbf{R} = \mathbf{R}\widetilde{\delta\boldsymbol{\omega}}. \tag{4}$$

Physically, $\widetilde{\delta\mathbf{w}}$ represents infinitesimal spatial rotation superimposed to the rotation $\mathbf{R}$. $\delta\mathbf{w}$, which is also denoted as spatial spin variables, is related to the variation of the rotational vector through

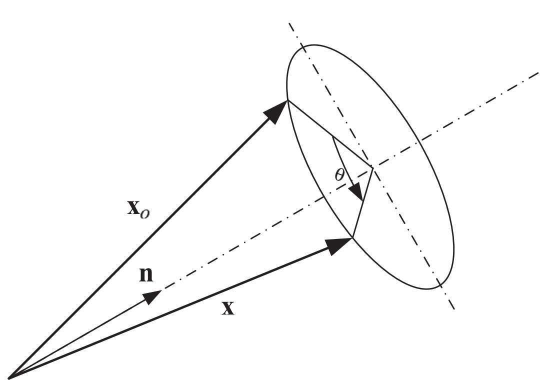

Fig. 1. Finite rotation of a vector.

$$\delta\mathbf{w} = \mathbf{T}_s(\boldsymbol{\theta})\,\delta\boldsymbol{\theta}, \tag{5}$$

with

$$\mathbf{T}_s(\boldsymbol{\theta}) = \mathbf{I} + \frac{1-\cos\theta}{\theta^2}\widetilde{\boldsymbol{\theta}} + \frac{\theta-\sin\theta}{\theta^3}\widetilde{\boldsymbol{\theta}}\,\widetilde{\boldsymbol{\theta}}. \tag{6}$$

The time derivative of the rotation matrix in spatial and material form is given by

$$\dot{\mathbf{R}} = \widetilde{\dot{\mathbf{w}}}\mathbf{R} = \mathbf{R}\widetilde{\dot{\boldsymbol{\omega}}}, \tag{7}$$

where the vectors $\dot{\mathbf{w}}$ and $\dot{\boldsymbol{\omega}}$ are spatial and material angular velocities, respectively.

The spatial and material quantities are connected by the relations

$$\delta\mathbf{w} = \mathbf{R}\,\delta\boldsymbol{\omega}, \tag{8}$$

$$\dot{\mathbf{w}} = \mathbf{R}\,\dot{\boldsymbol{\omega}}, \tag{9}$$

$$\ddot{\mathbf{w}} = \mathbf{R}\,\ddot{\boldsymbol{\omega}}. \tag{10}$$

The variation of the material angular velocity is given by (see [7])

$$\delta\dot{\boldsymbol{\omega}} = \dot{\delta\boldsymbol{\omega}} + \widetilde{\dot{\boldsymbol{\omega}}}\,\delta\boldsymbol{\omega}, \tag{11}$$

with $\dot{\delta\boldsymbol{\omega}}$ denoting the time derivative of the material spin variables.

3. Kinetic energy

In the present paper, the kinematic description proposed by Gruttmann et al. [1] is adopted (see Fig. 2). A beam with an arbitrary cross-section is considered. G and C are the centroid and the shear center of the cross-section. $\mathbf{e}_i$ and $\mathbf{a}_i$ $(i = 1, 2, 3)$ denote the global and cross-section-attached orthogonal coordinates systems, respectively. The transformation from the global coordinates system to the cross-section-attached one is defined by

$$\mathbf{a}_i = \mathbf{R}\,\mathbf{e}_i. \tag{12}$$

Let $\mathbf{x}_{\mathrm{P}}(x, y, z)$ denote the position vector of an arbitrary point P in the current configuration

$$\mathbf{x}_{\mathrm{P}}(x, y, z) = \mathbf{x}_{\mathrm{G}}(x) + y\,\mathbf{a}_2(x) + z\,\mathbf{a}_3(x) + \alpha(x)\,\overline{\omega}(y, z)\,\mathbf{a}_1(x), \tag{13}$$

with $\mathbf{x}_{\mathrm{G}}$ denoting the position vector of G in the current configuration.

The warping function $\overline{\omega}(y, z)$ is defined within the Saint–Venant torsion theory and refers to the centroid G, i.e.,

$$\overline{\omega} = \omega - y_c z + z_c y, \tag{14}$$

where ω refers to the shear center C with coordinates y_c, z_c. Note that in connection with ω, the following normality conditions hold

$$\int_A \omega\,\mathrm{d}A = 0, \quad \int_A \omega\,y\,\mathrm{d}A = 0, \quad \int_A \omega\,z\,\mathrm{d}A = 0. \tag{15}$$

Using Eq. (12), the expression (13) can be put in the form

$$\mathbf{x}_{\mathrm{P}} = \mathbf{x}_{\mathrm{G}} + \mathbf{R}\,(\mathbf{X} + \alpha\,\mathbf{n}_{\overline{\omega}}), \tag{16}$$

with $\mathbf{X} = [0\ y\ z]^{\mathrm{T}}$ and $\mathbf{n}_{\overline{\omega}} = [\overline{\omega}\ 0\ 0]^{\mathrm{T}}$.

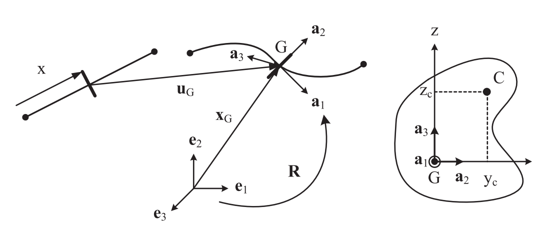

Fig. 2. Initial and current configuration of the beam.