Transportation Research Part E 101 (2017) 13–29

Contents lists available at ScienceDirect

Transportation Research Part E

journal homepage: www.elsevier.com/locate/tre

The multi-criteria constrained shortest path problem

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ARTICLE INFO

Article history:
Received 19 July 2016
Received in revised form 7 January 2017
Accepted 13 February 2017

Keywords:
Routing
Constrained shortest path
Multi-criteria
Pareto-optimal paths

ABSTRACT

In this study, we propose an exact method for finding all the Pareto-optimal paths for a multi-criteria constrained shortest path problem. We show that solving the special bi-criteria problem is equivalent to generating at most $|\mathbb{P}|$ constrained shortest paths with successive tightened constraints, where $|\mathbb{P}|$ is the total number of all Pareto-optimal paths. For the general multi-criteria case, we propose a decomposition procedure and theoretically prove that this method can identify all the Pareto-optimal paths from at most $(u-1)!|\mathbb{P}|$ candidate paths, where u is the number of criteria. Numerical studies demonstrate that our algorithm is highly efficient and robust.

1. Introduction

Finding the shortest path (SP) is a fundamental problem in transportation routing and logistics modeling. Due to problems of road congestion, environmental concerns, and traffic safety considerations, this problem is complicated by multi-criteria objective functions and various constraints. The best path, in many applications, cannot be measured by a single criterion such as cost, time, or environmental influence alone, but by some or all of them together. For example, IBM Global Procurement Center sets CO_2 emission together with cost and time as key performance indicators for its transportation and logistics operations. Many large shippers, such as HP, GE, and IKEA, consider environmental concerns in designing their supply chain networks (Wang et al., 2011), in which the best transportation routes do not mean the cheapest, fastest, or shortest. Furthermore, there are often constraints or targets in practice on resources and measures (e.g., cost and time), which are not part of the optimization criteria for the selection of a path. For example, the paths in the Beijing freeway network are acceptable only if they can satisfy flow balance constraints, side constraints, and unique link selection constraints (Wang et al., 2016).

1.1. Motivation

The classical multi-criteria shortest path (MCSP) problem without constraints can be found in transportation and logistics (Chang et al., 2005; Verma et al., 2012; Wang et al., 2013), telecommunications (Benyamina et al., 2012; Kumar and Banerjee, 2003), and scheduling (Gabrel and Vanderpooten, 2002). The MCSP problem is proven NP-hard (Serafini, 1986). There are studies of the constrained shortest path (CSP) problem (Karsten et al., 2015; Rivera et al., 2016; Salazar-González, 2014; Wang et al., 2016), which is proven NP-complete (Michael and David, 1979). When both multi-criteria objective functions

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http://dx.doi.org/10.1016/j.tre.2017.02.002
1366-5545/

and constraints need to be considered simultaneously, as required in many contemporary transportation settings described earlier, the problem becomes significantly complex. However, as shown in the literature review below, the Multi-Criteria Constrained Shortest Path (MCCSP) problem is not well studied.

MCCSP is a practically important problem owing to the fact that the majority of the problems encountered in industries, particularly in transportation and logistics, involve multi-objectives and are constrained in nature. This problem appears in a wide range of applications. For example, transportation and logistics exhibit hazardous impacts on the environment, such as energy consumption, land use, acidification, toxic effects on ecosystems and humans, noise, and the effect induced by greenhouse gas (GHG) emission. In hazardous material transportation and pollution-routing problems (Baykasoglu and Subulan, 2016; Bektas and Laporte, 2011; Demir et al., 2014; Verma et al., 2012), the route selection is often based on multiple criteria (e.g., time, cost, risk, distance, fuel, and GHG emission) and is subject to some hard constraints (e.g., link prohibition). In the container routing problem faced by shipping liners (Liu et al., 2016; Song et al., 2015; Wang et al., 2013), the routes need to satisfy many practical constraints (e.g., maritime cabotage) and various objectives (e.g., service level and cost). For traffic network problems (Wang et al., 2014; Guo and Yang, 2009), the system performance can be measured either in time unit by the total system travel time or in monetary unit by the total system travel cost; the performance is also subject to aggregate link flow constraints. The complexity is thus attributed to the varying nature of MCCSP problem. In this study, we propose an optimization approach and show the efficiency of our approach theoretically.

1.2. Related literature

On the MCSP problem side, a recent overview of research can be found in Jozefowiez et al. (2008). According to the authors, three approaches can be used to solve MCSP problem: *a priori* approach, *interactive* approach, and *a posteriori* approach. In the *a priori* approach, the decision-maker provides preferences for the different objectives. In the *interactive* approach, the choices of the decision-maker are made during the problem solving process. Finally, in the *a posteriori* approach, a set of potentially non-dominated solutions is generated first and then the decision-maker chooses among these solutions. The approach followed in this study can be categorized as an *a posteriori* approach.

Overviews of methods, which can be used to solve MCSP problem, can be found in Ehrgott (2005), Ehrgott and Gandibleux (2004), and Jozefowiez et al. (2008). These methods can be divided into scalar and Pareto methods. The scalar method often uses a weighted objective function. The advantage of this method is that the problem can be transformed to a single-objective problem, and existing (meta-) heuristics may thus be used (Jozefowiez et al., 2008). However, the disadvantage is that agreeing on a set of weights is not straightforward (Braekers et al., 2014). Contrary to the scalar method, the Pareto method uses the notion of Pareto dominance. The exact methods for identifying the entire set of Pareto-optimal paths can be categorized into labeling and ranking algorithms. The labeling algorithm is first studied by Hansen (1980) and Martins (1984). The algorithm can be regarded as an adaptation of Dijkstra's algorithm for the single-objective SP problem (Ahuja et al., 1993). The extension of Martins (1984) can be found in Gandibleux et al. (2006), Guerriero and Musmanno (2001), and Skriver and Andersen (2000). The main idea of the ranking algorithm is to generate a large set of ranked paths according to single criterion first and then delete paths dominated by other criteria in the set. This method has been applied to Martins (1984) and Tsaggouris and Zaroliagis (2009). The exact method presented in this study is regarded as a ranking algorithm.

On the CSP problem side, researchers have proposed a variety of efficient algorithms to solve the CSP problem in recent decades (Wang et al., 2016). (1) Dynamic Programming. Witzgall and Goldman (1965) first used dynamic programming to solve the CSP problem with one side constraint. Joksch (1966) extended the approach to solve the CSP problem with multiple side constraints. (2) Label-setting algorithm. This algorithm can improve the dynamic programming with high computational efficiency (Aneja et al., 1983). (3) Lagrangian relaxation combined with K-shortest path algorithm (Handler and Zang, 1980). (4) Branch-and-Bound approach based on Lagrangian relaxation (Beasley and Christofides, 1989). Recently, researchers are interested in highly efficient algorithms for solving large-scale CSP problems, particularly SP with resource constraints (SPPRC). The standard approach for solving the SPPRC problem in practice is based on dynamic programming and has a pseudopolynomial complexity. Boland et al. (2006) applied a state-space augmenting approach to accelerate the computation time of a label setting algorithm for SPPRC. Santos et al. (2007) introduced an improved exact algorithm through defining an efficient search direction based on the K-shortest path algorithm. Pugliese and Gurriero (2013) addressed an elementary SP problem with forbidden paths; the problem could be formulated as a specific instance of the SPPRC problem and two different solution approaches were defined and implemented. In this study, an efficient K-shortest path algorithm is also considered.

When both multi-criteria objective functions and constraints need to be considered simultaneously, the problem has received very limited attention despite of its importance. Climaco et al. (2003) solved the bi-criteria CSP problem numerically using an adaptation of K-shortest path algorithm. The constraints in their problem are typical resource-constraints. However, their approach cannot find all the Pareto-optimal paths. Whether their approach can be extended to multi-criteria case is unclear as well. Gabrel and Vanderpooten (2002) used a two-phase approach. They generated all candidate paths in the first phase and then selected satisfactory paths using an interactive procedure in the second phase. However, this approach cannot guarantee that all non-dominated and feasible paths can be found. In this study, we propose an exact and efficient method for finding all the Pareto-optimal paths for a general MCCSP problem. All the Pareto-optimal solutions are desirable because they are an indication that the solution space is exactly presented in the Pareto set (i.e., Pareto frontier), which clearly shows the trade-offs between the total cost and other environmental concerns in transportation routing