Asia Pacific Management Review 21 (2016) 213–218

Contents lists available at ScienceDirect

Asia Pacific Management Review

journal homepage: www.elsevier.com/locate/apmrv

Suitable assignment of individuals to positions based on consensus formation

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A R T I C L E I N F O

Article history:
Received 4 June 2015
Accepted 19 July 2016
Available online 17 November 2016

Keywords:
Aggregating methods
Cross evaluation
Relative distance
Consensus formation
Assignment problem

A B S T R A C T

This paper investigates how to assign individuals to suitable positions effectively and fairly. Firstly, some related standard mathematical evaluation methods such as DEA, total ranking method based on DEA and consensus formation method are reviewed. Secondly, using these methods and combining them under multi-criteria and scenarios, in Ishii (2014) we proposed two allocation models to assign suitable individuals to suitable positions, i.e., a ranking data model with respect to each position and a scenario model. Modifying a previous paper and the relative distance method by Cook and Kress (1984), a new consensus method is then proposed and applied to these two models. These models are firstly transformed into a transportation problem with a special structure since our model assumes that the number of positions does not exceed that of individuals, that is, it includes the case in which the number of positions is insufficient to cover all individuals. The transportation problem is then transformed into the classic assignment problem. From an optimal solution of the assignment problem, an optimal allocation of individuals to positions is found for each model.

Finally the paper concludes by showing the results and discussing applicability of the model to other allocation problems and to other evaluation methods that use linguistic terms denoting the suitability of individuals to positions.

1. Introduction

For the ranking of candidates, one of the most familiar methods is to compare the weighted sum of the votes for each candidate after determining suitable weights for each alternative. For example, the MVP (most valuable player) in Japanese baseball is selected using the results of the votes by a number of baseball reporters, though weights are fixed. Mathematical evaluation methods for candidates date back more than two hundred years. Borda (1781) initially proposed the "Method of Marks" more than two hundred years ago to obtain an agreement among different opinions. His method is surely useful for evaluating consumer preferences for commodities in marketing, or in ranking social policies in political science, for instance. It is, however, difficult to determine a suitable *a priori* weight for each alternative. In this context, Cook and Kress (1990) formulated a measure to automatically decide on the total rank order weight in order to hold the advantage using the DEA (Data Envelopment Analysis) model. DEA was originated by Charnes, Cooper, and Rhodes (1978) and extended by Banker, Charnes, and Cooper (1984). The basic DEA models are known as CCR (Charnes, Cooper and Rhode) and BCC (Banker, Charnes and Cooper) after the authors' initials. Later, Green, Doyle, and Cook (1996) evolved the distance measure to make it possible to decide on the total rank order of all candidates. The distance measure is based on a different idea that individual preference for a set of candidates should be aggregated (see related works of Cook (2006), Cook and Kress (1990), Green et al. (1996)). Tanabe and Ishii (2007) extended the distance measure to construct a joint ballot model.

This paper considers the suitable allocation of individuals to positions by consensus formation models based on their ranking with respect to various kinds of positions. Consensus formation is one kind of group decision-making method that aggregates the opinions of members of a group. This situation may be seen when players are assigned to positions by a manager and/or coach in baseball, soccer, American football and so on, and in the personnel affairs of companies, where possible candidates are recommended for the position of general manager based on executives' opinions.

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Peer review under responsibility of College of Management, National Cheng Kung University.

http://dx.doi.org/10.1016/j.apmrv.2016.07.001
1029-3132/

Firstly, some related mathematical evaluation methods are reviewed briefly in Section Review of related mathematical evaluation methodology. Section Consensus formation method shows the standard consensus formation methods considered thus far. Section Some extended consensus making models proposes new consensus making models and corresponding assignment problems to find the suitable allocation of candidates to positions in these models. Finally, Section Conclusion discusses further research problems and concludes the paper.

2. Review of related mathematical evaluation methodology

2.1. Data Envelopment Analysis

Data Envelopment Analysis (DEA) is a mathematical evaluation method for measuring the efficiency of decision-making units (DMU) on the basis of the observed data practiced in comparable DMUs, such as public organizations, e.g. departments, governments, universities, libraries and basic units including hospitals, banking, and so on. DEA was originated by Charnes et al. (1978) and extended by Banker et al. (1984) and thus the basic DEA models are known as CCR and BCC, named after the authors' initials (see Banker et al. (1984), and Charnes et al. (1978)). For measuring the efficiency of a DMU, the virtual input and output aggregating inputs and output by weights are used, as follows:

Let an input vector of DMU j be $(x_{1j},x_{2j},\cdots,x_{mj})$ and output $(y_{1j},y_{2j},\cdots,y_{sj})$, that is, x_{ij} is the amount of i th kind of inputs, $i = 1,2,\ldots,m$ and $y_{\ell j}$ that of ℓ th kind of outputs, $\ell = 1,2,\ldots,s$. Inputs are aggregated by weights $v_1,v_2,\cdots,v_m$ and results in a virtual input $v_1x_{1j}+v_2x_{2j}+\cdots+v_mx_{mj}$. Also aggregating outputs by weights, $u_1,u_2,\cdots,u_s$ results in a virtual output $u_1y_{1j}+u_2y_{2j}+\cdots+u_sy_{sj}$. We then define

$$\textit{efficiency of DM } j = \frac{\textit{virtual output of DMU } j}{\textit{virtual input of DMU } j},\ \ j = 1,2,\ldots,n$$

These weights are decided by solving the following linear fractional programming problem FP_O for each DMU O, O = 1, 2,..,n..

$$\begin{aligned} &< FP_O > \max_{u_r,v_j} \quad \theta = \frac{u_1y_{1O}+u_2y_{2O}+\cdots+u_sy_{sO}}{v_1x_{1O}+v_2x_{2O}+\cdots+v_mx_{mO}} \\ &\textit{subject to } \frac{u_1y_{1j}+u_2y_{2j}+\cdots+u_sy_{sj}}{v_1x_{1j}+v_2x_{2j}+\cdots+v_mx_{mj}} \le 1, \quad j = 1,2,\ldots,n \qquad (1) \\ &v_1,v_2,\cdots,v_m \ge 0, \quad u_1,u_2,\cdots,u_s \ge 0, \end{aligned}$$

That is, in order to calculate an optimal value of FP_O, favorable weights for DMU O, O = 1, 2, …, n are used since the objective function of FP_O is the efficiency of DMU O. The efficiency of DMU O is then checked to see whether the optimal value of FP_O is less than 1 or not. DMU O is called efficient if the optimal value is 1 and inefficient if the value is less than 1. Further FP_O is transformed into the following linear programming problem LP_O setting the virtual input of DMU O to be 1 as in (2) and multiplying both sides of each constraint (1) $\frac{u_1y_{1j}+u_2y_{2j}+\cdots+u_sy_{sj}}{v_1x_{1j}+v_2x_{2j}+\cdots+v_mx_{mj}} \le 1$, by $v_1x_{1j}+v_2x_{2j}+\cdots+v_mx_{mj}$, $j = 1,2,\ldots,n$.

$$\begin{aligned} &< LP_O > \max_{u_r,v_j} \quad \theta = u_1y_{1O}+u_2y_{2O}+\cdots+u_sy_{sO} \\ &\textit{subject to } v_1x_{1O}+v_2x_{2O}+\cdots+v_mx_{mO} = 1 \qquad (2) \\ &u_1y_{1j}+u_2y_{2j}+\cdots+u_sy_{sj} \le v_1x_{1j}+v_2x_{2j}+\cdots+v_mx_{mj}, \\ &j = 1,2,\ldots,n, \ \ v_1,v_2,\cdots,v_m \ge 0, \ \ u_1,u_2,\cdots,u_s \ge 0 \end{aligned}$$

Based on the optimal value of, $FP_O(LP_O)$ DMU O is determined to be efficient if the value is 1 and inefficient if the value is less than 1.

2.2. Total ranking method based on DEA

The following voting and results of voting are considered:

Each of n individuals has k votes and votes for k candidates among m candidates according to his or her preference. That is, each person selects k candidates up to k ranks according to preference. For each candidate j, let the number of votes captured by the first place candidate be y_{j1}, y_{j2} in the second place, …,y_{jk} in k-th place. The aggregated weighted sum $\phi_{jj} \triangleq \sum_{\ell=1}^{k} w_{j\ell}y_{j\ell}$ of captured votes is then calculated by using weights $w_{j\ell}, \ell = 1,2,\ldots,k$ determined as an optimal solution of the following linear programming problem P_j (Charnes et al., 1978).

$$\begin{aligned} &P_j: \textit{Maximize } \phi_{jj} = \sum_{i=1}^{k} w_{ji}y_{ji} \\ &\quad \textit{subject to } \phi_{qj} = \sum_{i=1}^{k} w_{ji}y_{qi} \le 1, \ q = 1,2,\ldots,m, \\ &\qquad\qquad w_{j1} \ge w_{j2} \ge \cdots \ge w_{jk} \ge 0. \end{aligned}$$

It seems, however, that the condition $w_{j1}\ge w_{j2}\ge\cdots\ge w_{jk}\ge 0$ is not suitable since the difference between higher ranks should be more than that between lower ranks. Moreover if weights between rank t and t+1 are equal, the information will be lost, and if $w_{jk} = 0$, it is equivalent to the votes up to rank k−1 It is therefore replaced by the following condition

$$w_{j1} \ge 2w_{j2} \ge 3w_{j3} \ge \cdots \ge kw_{jk}, \ \ w_{jk} \ge \frac{1}{(k+\cdots+1)\times k} \qquad (3)$$

(see Noguchi and Ishii (2000)) and the following linear programming problem $\overline{P}_j$ is considered.

$$\overline{P}_j: \textit{maximize } \ \phi_{jj} = \sum_{i=1}^{k} w_{ji}y_{ji}$$

$$\textit{subject to } \ \phi_{qj} = \sum_{i=1}^{k} w_{qi}y_{ji} \le 1, \ \ q = 1,2,\ldots,m$$

$$w_{j1} \ge 2w_{j2} \ge 3w_{j3} \ge \cdots \ge kw_{jk}, \ \ w_{jk} \ge \frac{1}{(k+\cdots+1)\times k}$$

Note that $\overline{P}_j$ gives a favorable set of weights to candidate $j = 1,2,\ldots,m$.

The following table is constructed from the optimal values of these linear programming problems. In Table 1, ϕ_{qj}^*, $q = 1,2,\ldots,m$ means the total score of candidate q using the favorable set of weights to candidate j,$j = 1,2,\ldots,m$, that is, the optimal solution (w_{ji}^*) of $\overline{P}_j$ and so $\phi_{qj}^* = \sum_{i=1}^{k} w_{ji}^* y_{qi}$.

Based on Table 1, we calculate the geometric mean $\overline{\phi}_i = \sqrt[n]{\phi_{i1}^*\phi_{i2}^*\cdots\phi_{im}^*}$ for each candidate i, i = 1, 2,…, m.

Candidates are ranked according to the value of the geometric mean, that is, the candidate with the greatest value will be in the first rank, the next greatest, the second, and so on. This ranking method is called cross evaluation (see Green et al. (1996)). For more

Table 1
Cross evaluation.

	Candidate 1	Candidate 2	…	Candidate m
Candidate 1	ϕ_{11}^*	ϕ_{12}^*	…	ϕ_{1m}^*
…	ϕ_{21}^*	ϕ_{22}^*	…	ϕ_{2m}^*
…	⋮	⋮	…	…
…	⋮	⋮	…	…
…			…	
…				
Candidate m	ϕ_{m1}^*	ϕ_{m2}^*	…	ϕ_{mm}^*