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Harmonic response calculation of viscoelastic structures using classical normal modes: An iterative method

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An efficient iterative method, which only requires normal modes, is presented to calculate the harmonic response of viscoelastic structures. The method only needs to iteratively solve a diagonal dynamic equation instead of solving the dynamic equation directly such that it takes $O(N^2)$ instead of $O(N^3)$. However, the iterative procedure based on lower normal modes cannot be converged to the exact result. A modal correction technique is therefore introduced to improve the accuracy of iterative results. Finally, the efficiency and applicability of the method are illustrated in terms of sandwich plates with different types of viscoelastic core.

1. Introduction

The structure with viscoelastic damping treatment may be one of the most effective ways for structural vibration reduction and noise control [1]. Increasing the use of structural systems with viscoelastic damping treatment (such as composite structural materials, active control and damage tolerant systems in airplane, rocket, spacecraft, satellite, ships and automobiles), the need to consider the dynamic analysis of viscoelastic structures is more than ever before. Harmonic response analysis is usually used to analyze the response of a structure subject to steady-state oscillatory excitation and therefore is of fundamental importance. The harmonic frequency responses are of interest in the dynamic problems of mechanical and structural systems subjected to harmonic loading that may be caused by reciprocating or rotating machine parts (such as compressors, fans, forging hammers and motors). Harmonic analysis plays a very important role in many areas such as finite element (FE) model updating, vibration and noise control, system identification, structural damage detection and dynamic optimization [2].

The equations of motion of linear viscoelastic structures with zero initial conditions can be expressed as

$$\mathbf{M}\ddot{\mathbf{u}}(t) + \int_0^t \mathbf{g}(t-\tau)\frac{\partial \mathbf{u}(\tau)}{\partial \tau}\mathrm{d}\tau + \mathbf{K}\mathbf{u}(t) = \mathbf{f}(t) \tag{1}$$

where $\mathbf{M}$ and $\mathbf{K} \in \mathbb{R}^{N\times N}$ are, respectively, the mass and stiffness matrices (assume the number of degrees of freedom (DOF) is N and only symmetric system matrices are considered), $\mathbf{f}(t)$ is the forcing vector and $\ddot{\mathbf{u}}(t)$ denotes the second-order time derivative of displacement vector $\mathbf{u}(t)$. Matrix $\mathbf{g}(t)$ consists of kernel functions, which are used in the literature under different names (such as after-effect functions, characteristic relaxation functions, retardation functions or heredity functions). Principally speaking, any causal model making the energy dissipation functional nonnegative, may be considered as a candidate for a damping model. It has been shown that [3,4], by choosing different kernel functions, the system can be reduced to different damping systems. In the special case, when $\mathbf{g}(t) = \mathbf{C}\delta(t)$ where $\mathbf{C}$ is a viscous damping matrix and $\delta(t)$ is the Dirac delta function, the system can be reduced to a familiar viscously damped system. Although these damped systems are physically different, they can be mathematically treated in a unified damping formula. Such damping model has been considered as the most generalized damping model within the scope of linear dynamic analysis [5]. An exhaustive study on the generalized damping model (including damping model, modal analysis, dynamic response and damping identification) may be found in Adhikari [6]. The equations of motion similar to Eq. (1) may arise in many different subjects, including ship dynamics [7], the energy dissipation in structural joints [8], vibration isolation [9,10], the dynamic of buildings [11,12], the noise control in automobiles and airplanes [1,13] or dynamic of railway track [14]. The governing dynamic responses of viscoelastic structures subjected to harmonic excitations, i.e., $\mathbf{f}(t) = \mathbf{F}\exp(\mathrm{i}\omega t)$ can be modeled using the following matrix problem:

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$$(-\omega^2\mathbf{M} + i\omega\mathbf{G}(i\omega) + \mathbf{K})\mathbf{U}(i\omega) = \mathbf{F} \tag{2}$$

where $\mathbf{G}(s) = L[\mathbf{g}(t)]$ with $s = i\omega$ and $L[\ ]$ denotes the Laplace transform. Since the forcing function is harmonic, the resultant steady-state response is also harmonic, i.e., $\mathbf{u}(t) = \mathbf{U}(i\omega)\exp(i\omega t)$ where $\mathbf{U}(i\omega)$ is the complex response (displacement) vector. It should be mentioned that viscoelastic structures can be directly modeled using Eq. (2) since the storage and loss modulus of viscoelastic materials can be directly obtained from experiments without any transformations (see [15–18] for details).

Generally speaking, two kinds of methods (direct frequency response method and mode superposition method) are usually used to calculate the harmonic responses of viscoelastic structures. The direct frequency response method (or so-called the direct method [19,20]) is based on the direct frequency results in an exact calculation by solving the inversion of the dynamic stiffness matrix directly at each frequency point. In the most likely case for many engineering applications, the direct frequency response method must be carried out for many frequency steps. Under such circumstance, it may be more effective by using the reduced basis technique, especially for large-scaled models. Often the complex modes are used as the reduced basis (i.e., so-called the modal superposition method), which allows us to treat the equilibrium equations as a reduced-order form such that the step-by-step solution is less costly. However, the solution of the complex modes of viscoelastic structures is time-consuming since the frequency-dependent damping matrix causes a nonlinear eigenproblem. Many studies are devoted to solving the nonlinear eigenproblem under different damping models, which can be split into state-space approaches [16,17,21–25] based on some internal variables and approximate approaches [5,26–31] based on normal modes. Recently, Cortés sand Elejabarrieta [32] suggested a harmonic analysis method for viscoelastically damped rod using the superposition of modal contribution functions. Abdoun et al. [19] presented a numerical method for forced harmonic vibration analyses of viscoelastic structures based on the asymptotic numerical method and a perturbation method. Martinez-Agirre and Elejabarrieta [20] developed a numerical method for the harmonic analysis of viscoelastic structures in terms of higher-order eigensensitivity technique. Bilasse et al. [33] discussed the complex modes based numerical method for the vibration problem of viscoelastic sandwich plates. Chazot et al. [34] studied the computation of harmonic response of viscoelastic multilayered structures in terms of a ZPST shell element. More recently, Bilasse and Oguamanam [35] presented a reduced-order method for the forced harmonic response analysis of large-scale sandwich plates with viscoelastic core. The reduced-order model is developed by projecting the original problem onto low-dimensional subspaces spanned by the real and complex modes of the viscoelastic structures. And the harmonic response can be obtained using the asymptotic numerical method in conjunction with automatic differentiation techniques. In general, the complex modal superposition method requires all the modes. Often it is difficult, or even unnecessary, to obtain all the eigenpairs of a large-scaled model, which means that the modal truncation scheme is generally used and the modal truncation error is therefore introduced. As a result, the quality of the calculated harmonic responses may be adversely affected. Li et al. [3,36] developed some correction modal methods for the harmonic analysis of viscoelastic structures to take into account the contribution of higher (unavailable) modes in terms of the lower modes and system matrices based on the Neumann expansion-series. However, these methods calculate the harmonic response using the superposition of complex modes.

As we know, the complex modes can be used to exactly calculate the harmonic response, but they are usually difficult to be exactly obtained, especially for viscoelastic structures. In this paper, to avoid using the complex modes, an efficient iterative method, which only requires the classical normal modes, is presented to calculate the harmonic response of viscoelastic structures. With the iterative method, the complex eigensolutions are avoided such that classical eigensolution algorithms are only involved and the harmonic analysis of large-scaled viscoelastic structures can be carried out efficiently. In the iterative method, the matrix consisting of non-diagonal element of the modal damping matrix is moved to the right-hand side, i.e., the method solves a diagonal dynamic equation instead of solving the dynamic equation directly. The convergence condition of the iterative procedure is also given. It will be shown that, when all the normal modes are available and the convergence condition is satisfied, the iterative method can be converged to the exact result. Since the modal truncation problem is involved, the iterative procedure based on lower normal modes cannot be converged to the exact result. Therefore, we introduce a modal correction technique to improve the iterative result. Finally, various numerical tests of harmonic forced vibration of sandwich plates with various viscoelastic models, shapes and boundary conditions are performed to validate the accuracy and efficiency of the iterative method.

2. Response calculation using complete normal modes

In this section, an iterative method using complete normal modes will be presented, and the convergence condition of it will be also given. It will be shown that the iterative method can be converged to the exact result if the convergence condition is satisfied.

2.1. Classical normal modes

The natural frequencies and corresponding modes (normal modes) can be obtained by solving the following eigenproblem

$$\mathbf{K}\boldsymbol{\varphi}_j = \omega_j^2\mathbf{M}\boldsymbol{\varphi}_j \quad \text{for } j = 1, 2, \ldots, N \tag{3}$$

where $\boldsymbol{\varphi}_j$ denotes the normal mode corresponding to the jth frequency ω_j. The procedure to form the stiffness matrix $\mathbf{K}$ can be seen in Li et al. [3,4]. We form the modal matrix $\boldsymbol{\Psi}$ as

$$\boldsymbol{\Psi} = [\,\boldsymbol{\varphi}_1, \quad \boldsymbol{\varphi}_2, \quad \ldots, \quad \boldsymbol{\varphi}_N\,] \tag{4}$$

These normal modes satisfy the orthogonality relationship over the mass and stiffness matrices and can be normalized such that

$$\boldsymbol{\Psi}^T\mathbf{M}\boldsymbol{\Psi} = \mathbf{I}_N \text{ and } \boldsymbol{\Psi}^T\mathbf{K}\boldsymbol{\Psi} = \begin{bmatrix} \ddots & & \\ & \omega_j^2 & \\ & & \ddots \end{bmatrix} = \boldsymbol{\Lambda} \text{ for } j = 1, 2, \ldots, N \tag{5}$$

Here $\mathbf{I}_N$ denotes the $(N \times N)$ identity matrix. The condition that the damping matrix must satisfy to diagonalize Eq. (2), is known as the proportional damping condition. These proportional damping conditions for viscoelastically damped systems given by Adhikari [37] take the following forms:

$$\mathbf{K}\mathbf{M}^{-1}\mathbf{g}(t) = \mathbf{g}(t)\mathbf{M}^{-1}\mathbf{K}$$
$$\mathbf{M}\mathbf{K}^{-1}\mathbf{g}(t) = \mathbf{g}(t)\mathbf{K}^{-1}\mathbf{M}$$

and

$$\mathbf{M}\mathbf{g}^{-1}(t)\mathbf{K} = \mathbf{K}\mathbf{g}^{-1}(t)\mathbf{M}$$

The proportional damping conditions are not usually encountered due to the fact that structures involving viscoelastic materials present a non-uniform damping distribution. Therefore, the concern of