Computers and Structures 133 (2014) 64–78

Contents lists available at ScienceDirect

Computers and Structures

journal homepage: www.elsevier.com/locate/compstruc

Explicit frequency response functions of discretized structures with uncertain parameters

G. Muscolino [a,*], R. Santoro [a], A. Sofi [b]

[a] Dipartimento di Ingegneria Civile, Informatica, Edile, Ambientale e Matematica Applicata, University of Messina, Villaggio S. Agata, 98166 Messina, Italy

[b] Dipartimento di Ingegneria Civile, dell'Energia, dell'Ambiente e dei Materiali (DICEAM), University "Mediterranea" of Reggio Calabria, Via Graziella, Località Feo di Vito, 89124 Reggio Calabria, Italy

ARTICLE INFO

Article history:
Received 27 August 2012
Accepted 19 November 2013
Available online 25 December 2013

Keywords:
Frequency response function
Uncertain parameters
Explicit solutions
Improved interval analysis
Upper bound and lower bound

ABSTRACT

A novel procedure for deriving approximate explicit expressions of the *frequency response function* (*FRF*) matrix of linear discretized structures with uncertain parameters is presented. The following main steps are required: (i) to decompose the deviation of the structural matrices with respect to their nominal values as sum of rank-one matrices; (ii) to derive the so-called *Rational Series Expansion* (*RSE*) which provides an approximate explicit expression of the *FRF* holding for any uncertainty model. The potentials of the *RSE* are demonstrated within the interval framework by determining the region of the modulus of the *FRF* of structures with uncertain-but-bounded parameters.

1. Introduction

In Structural Dynamics, the response can be evaluated in two dual domains, often referred to as time domain and frequency domain. In the time domain, the response is described by the amplitude vs. time and the main purpose of the dynamic analysis is the evaluation of the response maxima. Conversely, in the frequency domain, the response is described in terms of the modulus and phase vs. the circular frequency. In this context, the main operator is the *frequency response function* (*FRF*), also called *transfer function*.

The *FRF* is a complex function able to provide information about the behavior of a structure over a range of frequencies. For instance, the frequency domain response of an oscillator is evaluated simply multiplying the *FRF* by the Fourier transform of the forcing function. For multi-DOFs structural systems, the *FRF* describes the relationship between a local excitation applied at one location on the structure and the resulting response at another and/or the same location. It follows that the frequency domain approach often gives information useful for structural design purposes that cannot be alternatively caught by the time domain approach. Moreover, it is sometimes more convenient to perform the analysis in the frequency domain; as an example, for structures with frequency-dependent parameters or subjected to stationary random processes and so on.

Standard structural analysis tools, either in the time or frequency domain, are devoted to the numerical evaluation of the system response due to external loads for given geometry and material properties. However, in practical engineering problems, material properties, geometry and boundary conditions of a structure may experience fluctuations due to measurement and manufacturing errors or other factors. The variability in the structural parameters may significantly affect the response. Although such variability is often inherent in the material properties and in the layout of the structure, it could also be introduced by the analyst to get an optimal design or for identification purpose.

The uncertainties are usually described following two different points of view, known as probabilistic and non-probabilistic approaches. The probabilistic approach requires a wealth of data, often unavailable, to define the probability density function of the uncertain structural parameters. If available information is fragmentary or incomplete, non-probabilistic approaches, such as convex models, fuzzy set theory or interval models [1], can be alternatively applied. Among non-probabilistic approaches, the interval model may be considered as the most attractive analytical tool. This model, stemming from the so-called interval analysis [2–5], turns out to be the most suitable approach when the upper and lower bounds of a non-deterministic property are well defined but information on the type of the distribution is missing.

In the framework of probabilistic approaches, the *FRF* has been evaluated by Falsone and Ferro [6,7] in explicit form by taking into account the properties of the natural deformation modes of finite

* Corresponding author.
E-mail addresses: gmuscolino@unime.it (G. Muscolino), roberta.santoro@unime.it (R. Santoro), alba.sofi@unirc.it (A. Sofi).

0045-7949/$ - see front matter © 2013 Elsevier Ltd. All rights reserved.
http://dx.doi.org/10.1016/j.compstruc.2013.11.007

element discretized structures. Manan and Cooper [8] developed a probabilistic method to determine the uncertainty bounds of *FRFs* by using the Polynomial Chaos Expansion technique.

Within a non-probabilistic context, Moens and Vandepitte [9] proposed an interval finite element (*IFE*) approach to efficiently calculate close outer approximations on the *FRF* envelopes of structures with interval uncertainties. The *FRF* of systems with uncertain-but-bounded parameters was also evaluated by Manson [10] employing both the complex interval analysis and the complex affine arithmetic. De Munck et al. [11] proposed a response surface based hybrid (global optimization and interval arithmetic) procedure to predict the bounds of the *FRFs* of models with interval inputs using the interval and fuzzy finite element method. An *IFE* method for evaluating the *FRF* has been recently developed by Yaowen et al. [12] by extending to dynamic problems the element-by-element technique previously proposed by Muhanna and Mullen [13] within a static setting. Muscolino and Sofi [14] derived an approximate explicit expression of the *FRF* matrix to perform the stochastic analysis of truss structures with uncertain-but-bounded parameters subjected to stationary multi-correlated Gaussian random processes.

The aim of the present paper is to derive an approximate explicit expression of the *FRF* matrix of arbitrary discretized structures with uncertain parameters by properly generalizing the approach presented in Ref. [14]. The proposed procedure requires the following main steps: (i) the decomposition of the deviation of the mass, stiffness and damping matrices with respect to their nominal values as sum of rank-one matrices; (ii) the derivation of the so-called *Rational Series Expansion* (*RSE*) which provides an approximate explicit expression of the *FRF* of structural systems with uncertain parameters.

A remarkable feature of the proposed *RSE* of the *FRF* is that it holds for arbitrary discretized structures regardless of the model assumed for the uncertain parameters, either probabilistic or non-probabilistic. For the sake of generality, in the paper, first the *RSE* is derived as an explicit function of the fluctuating structural parameters without introducing any assumption on the model describing the variability of such parameters. Then, the *RSE* is applied within a non-probabilistic context by assuming that the uncertain parameters are bounded by intervals. Closed-form expressions of the bounds of the modulus of the interval *FRFs* of structures with uncertain-but-bounded parameters are derived through the joint use of the *RSE* and the so-called *improved interval analysis*, recently presented in Ref. [14,15] to limit the overestimation affecting the *"ordinary" or classical interval analysis* [2] due to the *dependency phenomenon* [5,13,16].

The accuracy of the proposed *RSE* of the *FRF* is demonstrated through numerical results concerning both truss and frame structures with uncertain material and geometrical properties.

2. Discretized structures with uncertain parameters

2.1. Equations of motion

The equations of motion of a quiescent n-DOF classically damped linear structure with uncertain properties subjected to the forcing vector $\mathbf{f}(t)$ can be cast in the form:

$$\mathbf{M}(\boldsymbol{\alpha})\ddot{\mathbf{u}}(\boldsymbol{\alpha},t)+\mathbf{C}(\boldsymbol{\alpha})\dot{\mathbf{u}}(\boldsymbol{\alpha},t)+\mathbf{K}(\boldsymbol{\alpha})\mathbf{u}(\boldsymbol{\alpha},t)=\mathbf{f}(t), \tag{1}$$

where $\mathbf{M}(\boldsymbol{\alpha})$, $\mathbf{C}(\boldsymbol{\alpha})$ and $\mathbf{K}(\boldsymbol{\alpha})$ are the $n\times n$ mass, damping and stiffness matrices of the structure which depend on the dimensionless uncertain parameters collected in the vector $\boldsymbol{\alpha}=[\alpha_1,\ \alpha_2,\ \ldots,\ \alpha_r]^T$ of order r, with the apex T meaning transpose; $\mathbf{u}(\boldsymbol{\alpha},t)$ is the vector of displacements and a dot over a variable denotes differentiation with respect to time t. The Rayleigh model is herein adopted for the damping matrix, i.e.:

$$\mathbf{C}(\boldsymbol{\alpha})=c_0\mathbf{M}(\boldsymbol{\alpha})+c_1\mathbf{K}(\boldsymbol{\alpha}), \tag{2}$$

where c_0 and c_1 are the Rayleigh damping constants having units s^{-1} and s, respectively.

The j-th element of the vector $\boldsymbol{\alpha}$ denotes the dimensionless fluctuation α_j of the j-th uncertain parameter d_j with respect to its nominal value $d_{0,j}$, i.e. $d_j = d_{0,j}(1+\alpha_j)$. In structural engineering applications, such fluctuations can be reasonably assumed to satisfy the condition $|\alpha_j| < 1$, with the symbol $|\bullet|$ denoting absolute value. For instance, if the uncertain Young's modulus of the j-th structural element is expressed as $E_j = E_{0,j}(1+\alpha_j)$, with $E_{0,j}$ denoting the nominal value, the fluctuation α_j must satisfy the condition $|\alpha_j| < 1$ to yield always positive values of E_j.

It is worth mentioning that the relationship between the structural matrices, $\mathbf{M}(\boldsymbol{\alpha})$ and $\mathbf{K}(\boldsymbol{\alpha})$, and the vector $\boldsymbol{\alpha}$ is often linear. Nevertheless, when such relationship is nonlinear, it is always possible to make the mass, damping and stiffness matrices depend linearly on the uncertain parameters by applying a suitable variable transformation. As an example, in the case of truss structures with fluctuating length of the bars, L_i, since the stiffness matrix is proportional to $1/L_i$, the position $Q_i = 1/L_i$ can be introduced to obtain a linear dependency on Q_i [7].

Based on this concept, the $n\times n$ mass, stiffness and damping matrices, $\mathbf{M}(\boldsymbol{\alpha})$, $\mathbf{K}(\boldsymbol{\alpha})$ and $\mathbf{C}(\boldsymbol{\alpha})$, can be expressed as linear functions of the uncertain properties, i.e.:

$$\begin{aligned}
\mathbf{M}(\boldsymbol{\alpha}) &= \mathbf{M}_0+\sum_{j=1}^{r_M}\alpha_j\mathbf{M}_j;\\
\mathbf{K}(\boldsymbol{\alpha}) &= \mathbf{K}_0+\sum_{j=r_M+1}^{r_M+r_K}\alpha_j\mathbf{K}_j;\\
\mathbf{C}(\boldsymbol{\alpha}) &= \mathbf{C}_0+c_0\sum_{j=1}^{r_M}\alpha_j\mathbf{M}_j+c_1\sum_{j=r_M+1}^{r_M+r_K}\alpha_j\mathbf{K}_j,
\end{aligned} \tag{3a-c}$$

where $r_M + r_K = r$ and

$$\begin{aligned}
\mathbf{M}_0 &= \mathbf{M}(\boldsymbol{\alpha})|_{\boldsymbol{\alpha}=\mathbf{0}}; \quad \mathbf{M}_j=\frac{\partial}{\partial\alpha_j}\mathbf{M}(\boldsymbol{\alpha})\Big|_{\boldsymbol{\alpha}=\mathbf{0}};\\
\mathbf{K}_0 &= \mathbf{K}(\boldsymbol{\alpha})|_{\boldsymbol{\alpha}=\mathbf{0}}; \quad \mathbf{K}_j=\frac{\partial}{\partial\alpha_j}\mathbf{K}(\boldsymbol{\alpha})\Big|_{\boldsymbol{\alpha}=\mathbf{0}}.
\end{aligned} \tag{4a-d}$$

In the previous equations, $\mathbf{M}_0$, $\mathbf{K}_0$ and $\mathbf{C}_0 = c_0\mathbf{M}_0 + c_1\mathbf{K}_0$ denote the mass, stiffness and damping matrices of the nominal structural system (i.e. for $\boldsymbol{\alpha} = \mathbf{0}$), which are positive definite symmetric matrices of order $n\times n$; furthermore, $\mathbf{M}_j$ and $\mathbf{K}_j$ are positive semi-definite symmetric matrices of order $n\times n$. Without loss of generality, in Eq. (3) mass and stiffness uncertainties are assumed to be fully disjoint.

2.2. Frequency domain response

It is well-known that the so-called time domain analysis can be used to determine the response of any linear structural system to any arbitrary loading [17]. However, in some cases, such as for structures with frequency-dependent parameters or in presence of stochastic stationary excitations and so on, it is more convenient to perform the analysis in the so-called frequency domain. Moreover, the frequency domain approach often gives information useful for structural design purposes that cannot be caught in the time domain [18]. The present paper addresses the problem of the evaluation of the *FRF* of discretized structural systems when the model parameters are subjected to variations.