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Concurrent topology optimization of structures and their composite microstructures

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ABSTRACT

Different from the independent design of macrostructures or material microstructures, a two-scale topology optimization algorithm is proposed by using the bi-directional evolutionary structural optimization (BESO) method for the concurrent design of the macrostructure and its composite microstructure. It is assumed that the macrostructure is made of composite materials whose effective properties are calculated through the homogenization method. By conducting finite element analysis of both structures and materials, sensitivity numbers at the macro- and micro-scale levels are derived. Then, the BESO method is used to iteratively update the macrostructures and the composite microstructures according to the elemental sensitivity numbers at both scales. Some 2D and 3D numerical examples are presented to demonstrate the effectiveness of the proposed optimization algorithm. A variety of optimal macrostructures and optimal material microstructures have been obtained.

1. Introduction

Structural optimization is becoming increasingly important due to the limited material resources, environmental impact and technological competition, all of which demand lightweight, low-cost and high-performance structures. Structural topology optimization technique seeks to achieve the best performance of a structure while satisfying various constraints such as a given amount of material. Compared with size and shape optimizations, topology optimization provides much more freedom and allows the designer to create novel and highly efficient conceptual designs for structures. Over the last two decades, various topology optimization algorithms, e.g. homogenization method [1], solid isotropic material with penalization (SIMP) [2–4], evolutionary structural optimization (ESO) [5,6], and level-set technique [7,8] have been developed. Unlike the continuous density-based topology methods, the ESO/BESO methods represent the structural topology and shape with discrete design variables (solid or void) so that the resulting design gives a clear structural boundary [6,9]. ESO was originally developed based upon a simple concept of gradually removing redundant or inefficient material from a structure so that the resulting topology evolves towards an optimum [6]. A later version of the ESO method, namely the bi-directional ESO (BESO) method, allows not only removing materials, but also adding materials to the design domain [10,11]. It has been demonstrated that the current BESO method is capable of generating reliable and practical topologies for various types of structures with high computational efficiency [9,12].

Currently, topology optimization techniques are mainly used to solve one-scale design problems either for the optimal design of macrostructures to improve their structural performance or for the material design to develop new microstructures with prescribed or extreme properties [13–17]. The optimal design of a macrostructure assumes that the structure is composed of given materials which can be selected from the available material database. The material design assumes that the material is made of periodic base cells and the macroscopic effective properties of the heterogeneous material are homogenized (averaged) according to the microstructure of the base cell. The inverse problem is a typical topology optimization problem for the material design which seeks an optimal microstructure of the base cell with prescribed or extreme macroscopic properties [18]. Such a microstructural design approach greatly enriches available materials which can be selected for constructing macrostructures by virtue of material properties, so as to satisfy required performance specifications.

The material selection for macrostructures is a complex process which involves not only material properties but also the service conditions such as structure shape, applied loadings and boundary conditions etc. Instead of selecting materials, Huang et al. [19] formulated a two-scale topology optimization problem and directly designed the microstructures of cellular materials and composites for given shapes of macrostructures using the BESO method. Zhang

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and Sun [20] discussed the scale effect in two-scale topology optimization of cellular materials and structures. However, an ideal design of a macrostructure should be the structure which has an optimal macroscopic topology, and meanwhile, is composed of materials/composites with optimal microstructures. That is, we should concurrently design the topologies of a macrostructure and its material microstructure. Inspired by biological systems, Rodrigues et al. [21] proposed a hierarchical computational procedure by integrating the macrostructures with a series of local material microstructures using the continuum density-based method, and Coelho et al. [22] extended this hierarchical procedure to the three-dimensional elastic structures. However, it is impossible to find optimal microstructure point-to-point even using parallel computing techniques. By constraining the volume fractions at the macro-level and the micro-level separately, Liu et al. [23] conducted a concurrent topology optimization of materials and structures where the macrostructure is solely composed of a material. The method was extended by Yan et al. [24] for minimizing the compliance of 2D thermoelastic structure, and by Niu et al. [25] for maximizing structural fundamental frequency. Deng et al. [26] studied the multi-objective design of lightweight thermoelastic structures using the concurrent optimization technique to minimize the structural compliance and the thermal expansion of a certain surface simultaneously. Unfortunately, the continuum density-based method cannot absolutely preclude "grey areas" with intermediate densities in the structural topology. The material properties at "grey areas" are roughly estimated through the material interpolation scheme but their microstructures are still unknown.

This paper proposes a two-scale topology optimization approach based on the BESO method for concurrently designing structures and materials. Different from the continuum density-based method, BESO utilizing discrete design variables is more suitable for concurrent topology optimization of structures and materials because there is no need to assume any properties or microstructures for intermediate materials for finite element analysis. The layout of the paper is as follows. A two-scale concurrent optimization model is established and illustrated in Section 2. The homogenization of effective material properties and sensitivity analysis of both macrostructures and materials are presented in Section 3. The procedure for implementing the BESO method for the concurrent optimization of macrostructures and material microstructure is given in Section 4. Section 5 presents several 2D and 3D numerical examples to demonstrate the effectiveness of the proposed optimization algorithm. Concluding remarks are given in Section 6.

2. Concurrent optimization model

Consider a macrostructure with known boundary conditions and external forces as illustrated by Fig. 1(a). The macrostructure is composed of two-phase composite with microstructures (Fig. 1(b)) periodically repeated by the base cell (Fig. 1(c)). In Fig. 1(c), phase 1 with density ρ_1 and phase 2 with density ρ_2 are represented by green and grey respectively. It is assumed that phase 1 is stiffer and heavier than phase 2 ($E_1 > E_2$, $\rho_1 > \rho_2$). The optimization objective is to find the spatial optimal topologies for both the macrostructure and its material microstructure so that the resulting macrostructure has the best loading-carrying capability for a given total weight. Optimizations at the two scales will be integrated into one system and resolved concurrently. For such a two-scale optimization problem, there are two finite element models, namely the macro model for macrostructure and the micro model for the base cell of material. To seek the maximum stiffness (or minimum mean compliance) of the macrostructure, the concurrent topology optimization can be formulated as

$$\text{Find} \quad x_i, x_j \quad (i = 1, 2, \ldots, M; \quad j = 1, 2, \ldots, N)$$

$$\text{Minimize} : C(x_i, x_j) = \frac{1}{2}\sum_{i=1}^{M} \boldsymbol{U}_i^{\mathrm{T}} \boldsymbol{K}_i(x_i, x_j) \boldsymbol{U}_i \tag{1}$$

$$\text{Subject to} : \boldsymbol{K}(x_i, x_j)\boldsymbol{U} = \boldsymbol{F} \tag{2a}$$

$$m(x_i, x_j) - W_f^* m_0 = 0 \tag{2b}$$

$$x_i, x_j = 0 \text{ or } 1 \tag{2c}$$

where C denotes the mean compliance of the structure. $\boldsymbol{F}$ and $\boldsymbol{U}$ represent the external force vector and the nodal displacement vector of the structural at the macro level, respectively. $\boldsymbol{K}$ is the stiffness matrix of the macrostructure which can be assembled by the elemental stiffness matrix $\boldsymbol{K}_i$. M is the total number of finite elements in the macro structure. W_f^* is the prescribed weight fraction of the final design. x_i and x_j are the binary design variables for the macro and micro models, respectively. In the macro model, $x_i = 1$ represents a solid element (two-phase composite or uniform material) and $x_i = 0$ represents a void element. In the micro model, when an element is made of phase 1, $x_j = 1$ and when phase 2, $x_j = 0$. $m_0 = \sum_{i=1}^{M} V_i \rho_1$ is the reference weight of the structure when the whole design domain is fully filled with phase 1. The weight of the design, m, can be expressed by

$$m = \sum_{i=1}^{M} x_i V_i \overline{\rho}_i, \; (x_i = 0 \text{ or } 1) \tag{3}$$

where $\overline{\rho}_i$ is the density of a solid element in the macro model. It is related to the micro model through mass conservation as

$$\overline{\rho}_i = \frac{\sum_{j=1}^{N} V_j [x_j \rho_1 + (1 - x_j)\rho_2]}{V_i}, \quad (x_j = 0 \text{ or } 1) \tag{4}$$

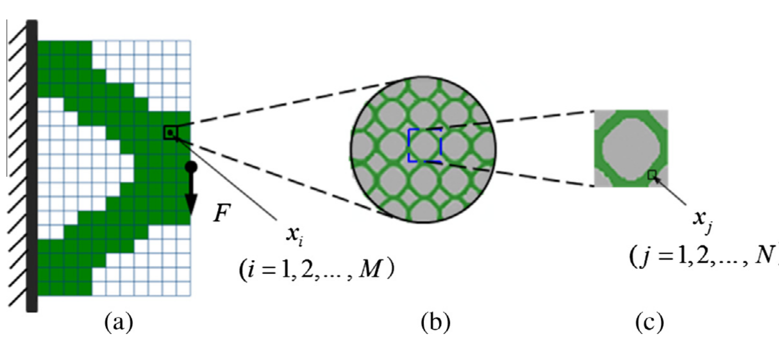

Fig. 1. A structure composed of composites: (a) macrostructure; (b) microstructure; (c) a base cell.