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The damping of a truss structure with a piezoelectric transducer

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Abstract

This paper re-examines the classical problem of active and passive damping of a piezoelectric truss. The active damping strategy is the so-called IFF (integral force feedback) which has guaranteed stability; both voltage control and charge (current) control implementations are examined; they are compared to resistive shunting. It is shown that in all three cases, the closed-loop eigenvalues follow a root-locus; closed form analytical formulas are given for the poles and zeros and the maximum modal damping. It is shown that the performances are controlled by two parameters: the modal fraction of strain energy v_i in the active strut and the electromechanical coupling factor k. The paper also examines the damping via inductive shunting and the enhancement of the electromechanical coupling factor by shunting a synthetic negative capacitance.

In the second part, a numerical example is examined and the analytical formulae are compared with predictions based on more elaborate models, including a full FE representation of the truss, the transducer, the electrical network and the controller. The limitations of the analytical formulae are pointed out.

Keywords: Piezoelectric transducers; Piezoelectric truss; Active and passive damping; Integral force feedback; Voltage control; Charge control; Resistive shunt; Inductive shunting; Negative capacitance

1. Introduction

The active damping of a truss with piezoelectric struts has been largely motivated by producing large, lightweight spacecrafts with improved dynamic stability; this classical problem has received a lot of attention over the past 15 years and very effective solutions have been proposed (e.g. [1]). One of them known as integral force feedback (IFF) is based on a collocated force sensor and has guaranteed stability [2].

Traditionally, the piezoelectric actuators have been controlled with a voltage amplifier; this is known to lead to substantial hysteresis caused by the ferroelectric behavior of the material, which requires an external sensor and closed-loop control for precision engineering applications. On the contrary, charge control allows to achieve a nearly linear relationship between the driving electrical value and the free actuator extension (e.g. [3]). The theory of IFF with charge control, and its implementation with a current amplifier, was reexamined in [4].

For space applications, because of the inherent constraints of the launch loads, the space environment, and the impossibility of in-orbit maintenance, there is a strong motivation to reduce or eliminate the power electronics associated to the piezoelectric actuators as well as the complex electronics associated to sensing (particularly in the sub-micron range where the sensor sensitivity becomes an issue). This has motivated the use of passive electrical networks as damping mechanisms [5–7]. The efficiency of such a damping mechanism depends very much on the ability to transform mechanical (strain) energy into electrical energy, that is to transfer strain from the vibrating structure to the transducer material, and to transform the strain energy into electrical energy inside the active material; the latter

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is measured by the *electromechanical coupling factor* k. Recent improvements have led to piezoelectric materials with coupling factors of $k_{33} = 0.7$ and more, making them a very attractive option for damping passively space structures. This paper compares the passive and the active options; they are presented in a very similar formalism and closed-loop results are presented, which allow a direct evaluation of the performances in terms of two physical parameters: the *modal fraction of strain energy* ν_i which controls the ability of every vibration mode to concentrate the strain energy into the transducer element, and the electromechanical coupling factor which measures the material ability to transform mechanical strain energy into electrical energy. Furthermore, the electromechanical coupling factor can be increased actively by shunting the piezoelectric transducer with a synthetic negative capacitance, as proposed by Forward [14] and demonstrated in [11].

Inductive shunting was first proposed in [5]; if the piezoelectric transducer is shunted on a RL circuit such that the natural frequency of the electrical circuit is tuned on the natural frequency of one mode, the system behaves like a tuned mass damper [7]. Remarkable performances can be achieved if the shunt parameters are perfectly tuned, but they drop rapidly when the natural frequency drifts away from its design value. The extension to multiple modes is addressed in [8], where the use of a set of parallel shunts is suggested; other methods are reviewed in [9].

More recently, promising alternative methods based on state switching have been proposed. The transducer is connected to a solid-state switch device which discharges periodically the piezoelectric element on a small inductor, producing a voltage inversion [16]. Nonlinear techniques will not be addressed in this paper.

2. Governing equations

Consider the linear structure of Fig. 1, assumed undamped for simplicity, and equipped with a discrete piezoelectric transducer of the stacked design (d_{33}). The stack includes n disks; its stiffness with short-circuited electrodes is K_a and its unloaded capacitance is C. The structure is defined by its mass matrix M and its stiffness matrix K (excluding

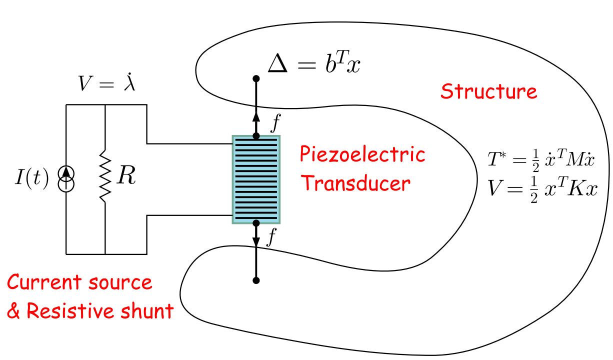

Fig. 1. Linear structure equipped with a piezoelectric transducer, a current source and a shunted resistor.

the transducer). The dynamics of this system can be handled with the Lagrange equations.

Using a flux linkage formulation for the electrical quantities, the Lagrangian of the system reads

$$L = T^* + W_e^* - V, \tag{1}$$

where

$$T^* = \frac{1}{2}\dot{x}^{\mathrm{T}} M \dot{x} \tag{2}$$

is the kinetic coenergy of the structure,

$$V = \frac{1}{2}x^{\mathrm{T}} K x \tag{3}$$

is the strain energy in the structure, excluding the piezoelectric transducer, and

$$W_e^*(\dot{\lambda}) = \frac{1}{2}C(1-k^2)\dot{\lambda}^2 + nd_{33}K_a\dot{\lambda}\Delta - \frac{1}{2}K_a\Delta^2 \tag{4}$$

is the coenergy function of the piezoelectric transducer [10]. In (4), $\dot{\lambda} = V$ is the voltage at the electrodes of the transducer, $\Delta = b^{\mathrm{T}}x$ is its total extension, d_{33} the piezoelectric constant and n the number of disks in the piezo stack (the free extension is $\delta = nd_{33}V$). Note that the constitutive equations of the transducer follow from

$$Q = \frac{\partial W_e^*}{\partial \dot{\lambda}}, \quad f = -\frac{\partial W_e^*}{\partial \Delta}. \tag{5}$$

One finds

$$\begin{Bmatrix} Q \\ f \end{Bmatrix} = \begin{bmatrix} C(1-k^2) & nd_{33}K_a \\ -nd_{33}K_a & K_a \end{bmatrix} \begin{Bmatrix} V \\ \Delta \end{Bmatrix}. \tag{6}$$

The virtual work of the non-conservative forces is

$$\delta W_{\mathrm{nc}} = I\delta\lambda - \frac{\dot{\lambda}}{R}\delta\lambda + F\delta x, \tag{7}$$

where I is the current source intensity and F is the vector of external forces applied to the structure. We assume $F = 0$ in this discussion.

The Lagrange's equations relative to the generalized coordinates x and λ give respectively

$$M\ddot{x} + (K + K_a bb^{\mathrm{T}})x = bK_a nd_{33}V, \tag{8}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}[C(1-k^2)V + nd_{33}K_a b^{\mathrm{T}}x] + \frac{V}{R} = I, \tag{9}$$

where $V = \dot{\lambda}$. These two equations govern the system dynamics when a current source is used. When a voltage source is used instead (the shunted resistor becomes irrelevant in this case), λ ceases to be a generalized variable and Eq. (8) applies alone.

3. Active damping with IFF and voltage control

Eq. (8) is rewritten in Laplace form

$$Ms^2x + (K + K_a bb^{\mathrm{T}})x = bK_a\delta, \tag{10}$$