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Effects of random system properties on the thermal buckling analysis of laminated composite plates

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ABSTRACT

This paper examines the effect of random system properties on thermal buckling load of laminated composite plates under uniform temperature rise having temperature dependent properties using HSDT. The system properties such as material properties, thermal expansion coefficients and thickness of the laminate are modeled as independent random variables. A C^0 finite element is used for deriving the eigenvalue problem. A Taylor series based first-order perturbation technique is used to handle the randomness in the system properties. Second-order statistics of the thermal buckling load are obtained. The results are validated with those available in the literature and Monte Carlo simulation.

1. Introduction

The laminated composite plate is one of the important structural elements, which is widely used in a variety of high performance engineering systems including aircraft, submarine, automotive, naval and space structures. When the plate is subjected to temperature change, thermally induced compressive stresses are developed in the constraint plate due to thermo-elastic properties and consequently buckling takes place. Thin plate structure becomes unstable at relatively lower temperature and buckles in the elastic region. Hence, the thermal buckling represents an important parameter for consideration and plays the significant role in the design of the structures.

The structure made of composite materials have more randomness and variability in the system properties compared to conventional isotropic structures as a large number of parameters are involved with their fabrication and manufacturing processes. The uncertainties in the system parameters inevitably create uncertainty in the effective system properties, which finally lead to dispersion in the structural response. In deterministic analysis, the system uncertainties are neglected and only the mean response is given which obviously misses the deviation caused by the randomness in the system parameters.

For a reliable design especially for sensitive applications it is important for the designer to have accurate knowledge of the structural response, other wise the predicted response may differ significantly which leads to the structure unsafe. In this regard, systematic efforts have been made to quantify the uncertainty in analysis and design of the structures in the past. One can take a note of NESSUS software, which is a general-purpose, probabilistic analysis program that simulates variations and uncertainties in loads, geometry, material behavior and other user-defined inputs to compute probability of failure and probabilistic sensitivity measures of engineered systems. But, many more things are still to be done in this direction. In the present study, attempts have been made to handle the thermal buckling problem having temperature dependent material properties in random environments in the framework of higher order shear deformation theory which has not been addressed till today to the best of authors' knowledge.

Considerable researches based on the deterministic analysis have been done on the thermal buckling investigations of composite plates using either classical theory of plates, first-order shear deformation theory (FSDT) and higher order shear deformation theory (HSDT) subjected to uniform or non uniform temperature distribution with temperature independent material properties [1–12]. However, a limited number of literature are available with temperature dependent material properties. Notably among them are Chen and Chen [13], Shen [14], Srikanth and Kumar [15], Shariyat [16], and Pandey et al. [17].

Relatively little efforts have been made in the past by the researchers and investigators on the prediction of the thermal buckling response of the structures made of laminated composites plates having random system properties. However, no work is available dealing with thermal buckling of laminated composite plates with random geometric and temperature dependent properties. Handa and Anderson [18] presented a stochastic finite element method (SFEM), which estimates the mean values, standard

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deviations and correlation coefficients of structural displacement, and stresses by taking into account variation in applied loads, dimensions and material properties. Nakagiri et al. [19] studied simply supported (SS) laminated plate with the SFEM taking fiber orientation, layer thickness and number of layers as random variables, and found that the overall stiffness of fiber reinforced plastic laminated plates is found out to be largely dependent on the fiber orientation. Chen et al. [20] presented a probabilistic method to evaluate the effect of uncertainties in geometrical and material properties of structure on the random vibration response. Lin and Kam [21] investigated the buckling response of frame with random initial imperfections, uncertain section and material properties using stochastic finite element method via mean-centered second order perturbation technique. Englested and Reddy [22] studied metal matrix composites based on probabilistic micro mechanics nonlinear analysis. They used Monte Carlo simulation (MCS) with different probabilistic distributions to incorporate the uncertainty in basic material properties. Zhang and Ellinwood [23] analyzed the effect of material properties on the elastic stability of structural members and frames using stochastic finite element method. They used mean-centered first-order perturbation technique (FOPT) using FEM to evaluate the buckling load. Noor Ahmed et al. [24] predicted the variability in the nonlinear response of composite structures associated with variations in the geometric and material parameters of the structure. They used hierarchical sensitivity analysis to identify the major parameters and fuzzy set analysis to determine the range of variation of the response. Graham and Siragy [25] studied the variability of the random buckling load of beams and plates with stochastically varying material and geometric properties using the concept of the variability response function. Graham and Deodatis [26] studied the variability of random response displacements and eigenvalues of structures with multiple random material and geometric properties using variability response functions. Singh et al. [27,28] investigated the second order statistics of buckling load analysis of laminated composite plates/panel by stochastic classical approach and the FEM for cross-ply and angle-ply laminates using the FOPT. Van den Nieuwenhof and Coyette [29] investigated sensitivity analysis to the random parameters such as material and shape parameters using SFEM and independent MCS. Stefanou and Papadrakakis [30] computed response variability for the case of combined uncertain material (Young's modulus, Poisson's ratio) and geometric (thickness) properties using stochastic finite element analysis. Onkar et al. [31] used a generalized layer wise stochastic finite element formulation for the buckling analysis of homogeneous and laminated plates with random material properties using FOPT in conjunction with HSDT.

Most recently, the present authors investigated the effects of randomness in system properties on elastic buckling and free vibration of laminated composite plates resting on elastic foundation by assuming the system properties such as Young's modulus, shear modulus, Poisson's ratio and foundation parameters as random inputs variables using C^0 FEM in conjunction with FOPT based on the HSDT [32–34].

The present study investigates the thermal buckling characteristics of laminated composite plates in the presence of small random variability in the system properties in the framework of the higher order shear deformation theory. The elastic moduli, shear moduli and Poisson's ratios of the constituent materials, thermal expansion coefficients and lamina thickness of the plate are treated as independent random variables. A C^0 finite element approach with a mean-centered first-order perturbation technique is employed to determine the second-order statistics (mean and standard deviation) of the buckling temperature of laminated composite plates with and without temperature dependent properties. The numerical results present the effects of the material properties, the support conditions and the plate thickness on the mean buckling load and its dispersion with respect to various input random variables. The effect of plate parameters and lamination schemes on the thermal buckling temperature are also examined. It is found that there is a significant effect of the randomness in the system properties on the thermal buckling behavior of the composite laminates.

2. Mathematical formulation

Consider a rectangular laminated composite plate of length a, width b, and thickness h, which consists of N number of orthotropic layers as shown in Fig. 1. All orthotropic layers are of uniform thickness. The mid plane of the plate is considered as the reference plane. The thickness coordinate, z of the top and bottom surfaces of any (kth) layer is denoted by $z_{(k+1)}$ and z_k, respectively. The fibers of kth layer are oriented at an angle θ_k to the x-axis.

2.1. Displacement field model

In the present study slightly modified form of third order shear deformation theory as given in Refs. [35–37], which is also known as the higher order shear deformation theory is employed. The displacement field model [35–37] is slightly modified after incorporating zero transverse shear stress conditions at the top and the bottom surfaces of the plate. In the modified form, the derivatives of the out-of-plane displacement arising after imposing the said conditions are assumed as separate field variables. Thus, 5 field variables with C^1 continuity is transformed into 7 field variables, with C^0 continuity. Hence, the displacements along the x, y, and z axes for an arbitrary composite laminated plate are expressed as [38]

$$\begin{aligned} \bar{u} &= u + f_1(z)\psi_x + f_2(z)\theta_x; \\ \bar{v} &= v + f_1(z)\psi_y + f_2(z)\theta_y; \\ \bar{w} &= w; \end{aligned} \tag{1}$$

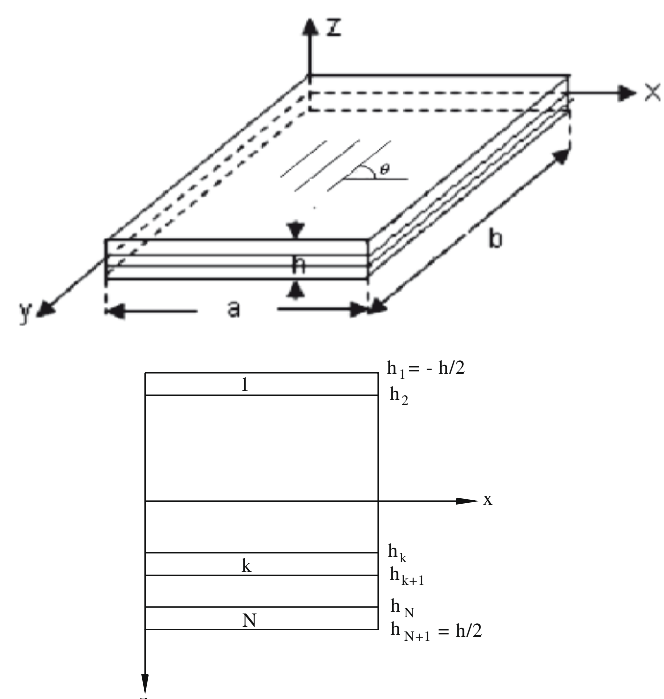

Fig. 1. Geometry of laminated composite plate.