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Component mode synthesis methods using partial interface modes: Application to tuned and mistuned structures with cyclic symmetry

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ABSTRACT

We present new component mode synthesis methods using partial interface modes which are the structure normal modes resulting from the static condensation of the structure to the interface between the substructures and which are possibly clamped at a part of this interface. These methods are the generalization of the classical component mode synthesis methods and those using the interface modes. These methods allow to reduce the number of the interface coordinates and at the same time to keep some of the physical interface displacements. These methods are applied to a structure with cyclic symmetry in both tuned and mistuned cases.

1. Introduction

Component mode synthesis (CMS) or dynamic substructuring methods consist in performing the dynamic analysis of structures by decomposing the structure into substructures and by projecting the equation of motion of each substructure on a projection basis to obtain the reduced systems of the substructures before performing the substructure coupling to obtain the reduced system of the whole structure. In the classical CMS methods, the substructure projection basis is composed, on one hand, by the normal modes of the substructure with various boundary conditions at the interface, such as fixed interface [18,19,36,39,40], free interface [3,14,18,20,32,36,37,49,50,61], hybrid interface [20,28,49,73,74], or loaded interface [6], and on the other hand, by Ritz vectors derived from the static deformation shapes commonly called the static modes, such as the constraint modes, the attachment modes, the residual attachment modes etc. CMS methods have been described in several text books [5,31,41,52,54], many insights, variants and improvements have been proposed [1,8,23,26,27,29,38,42–46,48,56,58–60,64,66,69–71], and CMS methods have been widely used for a large range of applications [4,7,9,10,15–17,22,24,30,35,47,51,53,55,62,63,65,67,76–80]. A history, review and classification of CMS methods can be found in [25].

In the classical CMS methods, the generalized coordinates associated with the static modes are in most of the cases the displacements at the interface between the substructures, leading to reduced systems with large size due to the important number of degrees of freedom (DOF) at the interface. In order to reduce the number of interface DOF, the CMS methods using interface modes has first been developed for the fixed interface CMS method [2,12,13,21] and then extended to the free and hybrid interface CMS methods [72]. In these methods, the static modes are replaced by the interface modes, also called the junction modes or the eigen modes of the Poincaré–Steklov operator, which are the first few normal modes of the whole structure after performing the Guyan static condensation [33] to the interface between the substructures. The interface displacements associated with the static modes in the classical CMS methods are then replaced by a few generalized coordinates associated with the interface modes. Alternative approaches for reducing the interface DOF were also proposed in [2,11,21,34].

Although the CMS methods using interface modes produce reduced systems with very small size, one drawback is that all the interface DOF are removed from the reduced system. The presence of a part of the interface DOF in the reduced system is however sometimes desirable and even essential, either because these DOF are not numerous and they can provide quick and useful information, or because one needs to deal directly with them while solving the reduced system, for example to impose prescribed motions or to take into account local non-linearities such as contact, friction or free-play. The aim of this paper, which is a continuation of the work in [72], is to develop new CMS methods using partial interface modes which fix this drawback. These methods allow at the same time an important reduction of the number of the

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interface DOF like in the CMS methods using interface modes, and the conservation of some interface DOF in the reduced system like in the classical CMS methods. To reach this aim, the approach in this work is that, instead of computing the interface modes, the latter are approximated by applying a second level CMS method on Guyan's reduced system resulting from the static condensation of the whole structure to the interface between the substructures. The DOF of Guyan's reduced system are partitioned into two sets containing respectively the interface DOF to be eliminated and those to be kept in the final reduced system, the former being considered as the interior DOF and the latter as the interface DOF in the second level CMS method. The choice of the kept interface DOF depends on the need of the user to keep them in the reduced system. The partial interface modes are defined as a first few normal modes of Guyan's reduced system in which some of the kept interface DOF can be clamped, depending upon which CMS method, i.e. with fixed, free or hybrid interface, is applied to Guyan's reduced system. The partial interface modes are completed with the static modes of Guyan's reduced system, whose associated generalized coordinates are precisely the kept interface DOF, and together they replace the interface modes or the substructure static modes in the projection basis. The classical methods and the methods using interface modes are particular cases of the new methods using partial interface modes, the former are obtained when all the interface DOF are kept, and the latter when all the interface DOF are eliminated.

This paper is organized as follows: In Section 2, the classical CMS methods and the methods using interface modes are reminded. In Section 3, the new CMS methods using partial interface modes are presented. Section 4 deals with the case of structures with cyclic symmetry. In Section 5, the new CMS methods are applied to compute the eigen frequencies and modes and the frequency response of a tuned and mistuned bladed disk, with several selections of partial interface modes and kept interface DOF. They are compared with the whole structure computations and also with the classical methods and the methods using interface modes.

2. Classical methods and methods using interface modes

2.1. Substructure description, reduced system and coupling

We consider a structure S which is decomposed into n_s substructures S_j $(j = 1, \ldots, n_s)$ which do not overlap. We denote by L^S the part of S which consists in the frontier between the substructures and by L_j the frontier of S_j with the adjacent substructures. L^S and L_j will be called the interface of S and S_j. The interface L^S is partitioned into L^S_k, the interface DOF to be kept in the final reduced coupled system, and L^S_e, the interface DOF to be eliminated. The number of DOF in L^S_k is very small compared to the number of DOF in L^S_e. The kept interface L^S_k is then partitioned into the fixed kept interface L^S_{ck} and the free kept interface L^S_{ak}.

For each substructure S_j, the interface L_j is also partitioned into the fixed interface L_c and the free interface L_a, thus L_j can be fixed ($L_a = \varnothing$ and $L_c = L_j$), free ($L_c = \varnothing$ and $L_a = L_j$) or hybrid ($L_c \neq \varnothing$, $L_a \neq \varnothing$, $L_j = L_c \cup L_a$), in the latter case S_j is supposed to be constrained when L_c is fixed. The choice of L_c and L_a can be different from one substructure to another, and it is completely independent of the choice of L^S_{ck} and L^S_{ak}.

The vectors of the physical displacements of S, L^S, L^S_e, L^S_k, L^S_{ck}, L^S_{ak}, S_j, L_j, L_c and L_a are respectively $\mathbf{x}^S$, $\mathbf{x}^S_L$, $\mathbf{x}^S_{Le}$, $\mathbf{x}^S_{Lk}$, $\mathbf{x}^S_{Lck}$, $\mathbf{x}^S_{Lak}$, $\mathbf{x}$, $\mathbf{x}_L$, $\mathbf{x}_{Lc}$ and $\mathbf{x}_{La}$. Let us define the boolean matrices $\mathbf{P}^S_{S_j}$ which restricts $\mathbf{x}^S$ to $\mathbf{x}$, $\mathbf{P}^L_{L_j}$ which restricts $\mathbf{x}^S_L$ to $\mathbf{x}_L$, and $\mathbf{P}_L$, $\mathbf{P}_c$ and $\mathbf{P}_a$ which restrict $\mathbf{x}$ to $\mathbf{x}_L$, $\mathbf{x}_{Lc}$ and $\mathbf{x}_{La}$ respectively: $\mathbf{x} = \mathbf{P}^S_{S_j}\mathbf{x}^S$, $\mathbf{x}_L = \mathbf{P}^L_{L_j}\mathbf{x}^S_L = \mathbf{P}_L\mathbf{x}$, $\mathbf{x}_{Lc} = \mathbf{P}_c\mathbf{x}$ and $\mathbf{x}_{La} = \mathbf{P}_a\mathbf{x}$.

The equilibrium equation of the isolated substructure S_j is written as:

$$\mathbf{K}\mathbf{x} + \mathbf{C}\dot{\mathbf{x}} + \mathbf{M}\ddot{\mathbf{x}} = \mathbf{f}_e - {}^t\mathbf{P}_L\mathbf{f}_L. \tag{1}$$

$\mathbf{K}$, $\mathbf{C}$ and $\mathbf{M}$ are the stiffness, damping and mass matrices of S_j, $\mathbf{f}_e$ are the external forces applied on S_j and $\mathbf{f}_L$ are the interface reactions exerted by S_j on L_j. The left superscript ${}^t(\,)$ denotes the transpose of a vector or a matrix.

The CMS methods consist in expressing the displacements of the substructure as a linear combination of the Ritz vectors in a projection basis $\mathbf{Q}$, i.e. $\mathbf{x} = \mathbf{Q}\mathbf{q}$, where $\mathbf{q}$ is the vector of the generalized coordinates. By projecting the equilibrium equation (1) on the projection basis $\mathbf{Q}$, we obtain a reduced system:

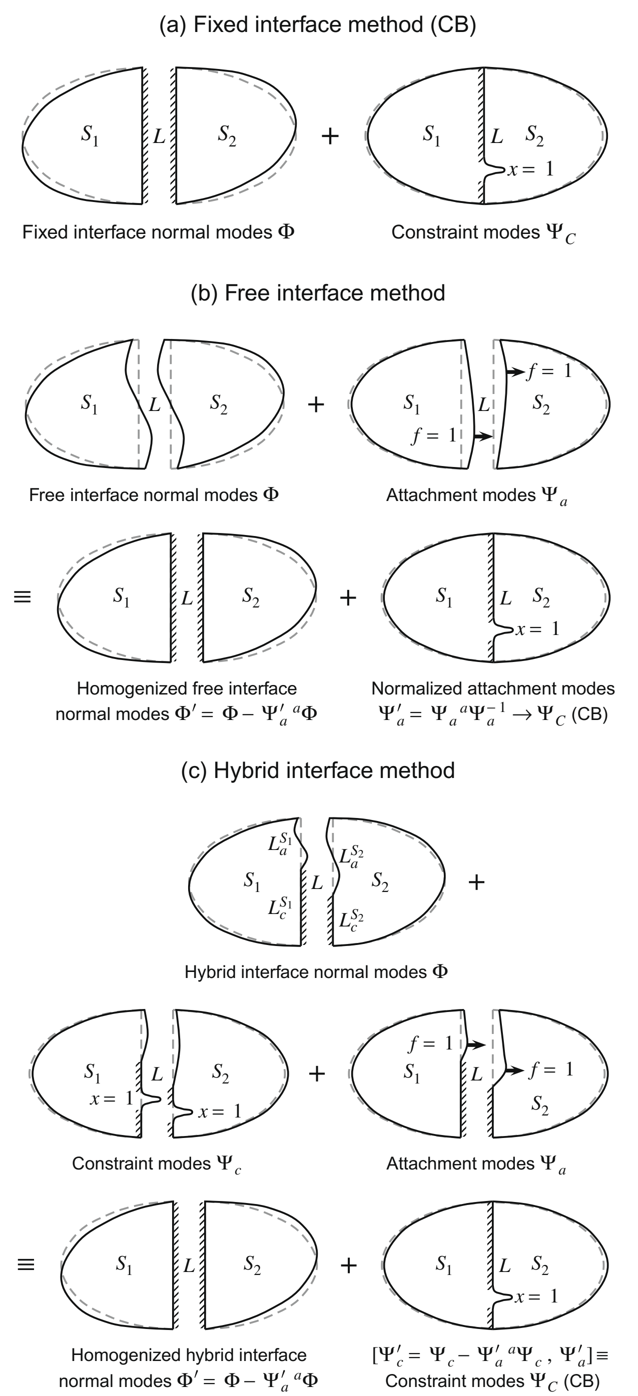

Fig. 1. Classical CMS methods.