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Optimal discrete piezoelectric patch allocation on composite structures for vibration control based on GA and modal LQR

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ABSTRACT

The optimization of piezoelectric patches allocation in composite structures is analysed in this paper. The finite element method and a linear quadratic regulator are used to study the electro-mechanical behaviour and the gain calculation. Due to the discrete nature of the problem, a simple binary Genetic algorithm is used as an optimization tool. Three examples are presented related to the optimal allocation based on Lyapunov functional. The PSD (Power Spectral Density) of the state space variables as well as input voltages are presented in order to identify the controlled modes and to show the effective attenuation obtained due to control of specific modes.

1. Introduction

In recent years, piezoelectric materials has been used in several areas and in a wide variety of problems related to vibration control, noise reduction or elimination, shape control and positioning control [1–5].

In vibration control, active control of lightweight structures may be the best solution due to the low damping (passive control). In any case, it is always desirable to perform the active control minimizing the control forces (which are constrained by the actuator force limits) and the attenuation of the structural vibration, although these two aspects lead to a conflicting task. The proper choice of the free parameters of the control algorithm as well as the positioning and number of piezoelectric actuators play the main role in achieving this goal. Modal control allows the rational use of the energy spent in control, since it is possible to control only the most important vibration modes (having the higher energy content) and neglecting the other ones.

Ning [6] presented an optimal design method with respect to the number and placement of piezoelectric patch actuators in active modal vibration control on a plate using a genetic algorithm (GA). This author used the eigenvalue distribution of the energy matrix of the control input force as the function to determine the optimal number and positions of the patches, concluding that the initial disturbance conditions is the key factor.

The total weighted energy method was proposed by Ang et al. [7] to obtain the weighting matrices for the modal LQR (linear quadratic regulator) control. The correct choice of the weighting matrices can generate vibration attenuations, which are proportional to the input voltages. Then, a compromise between vibration attenuation and input voltage should be obtained. A balanced vibration control and low input cost may be attained considering three design variables: the total kinetic energy, the strain energy and the input energy. The paper highlighted the advantages in using modal control analysis due to reduction of computational cost.

Roy and Chakraborty [8,9], Satpathy [10] and Chakraborty and Roy [11] presented the active vibration control of smart FRP (fibre reinforced polymer) composite plate and shell structures. They used a layered plate or shell finite element and an improved GA to optimize the positioning of piezoelectric patches and the weighting matrices $\boldsymbol{Q}$ and $\boldsymbol{R}$ for the control. The reasoning for choosing best allocation among patch positions was based on the damping ratio of the actual responses.

Based on a controllability index Ω, Wang and Wang [12,13] used a binary coded GA to find the optimal placement of a previously defined number of patches. Displacements and input voltages time histories are presented. As it is shown, in some cases the optimal solution differs from the intuitive positioning based on the mode shapes.

Several authors addressed the problem of allocation of the piezoelectric patches. Araújo et al. [2] used a Direct Multisearch Method with topology optimization of composite plates in order to reduce the modal loss factor by the co-located negative velocity feedback control. They apply the proposed methodology to a

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composite plate with coarse mesh refinement. Bachmann et al. [3] used a strain based optimization approach to allocate the patches. A MATLAB routine was also used allowing an exhaustive search of the best allocations for the patches (like a Monte Carlo sampling).

Potami [1] investigated three different approaches for the optimal positioning of sensors and actuators. All the proposed approaches present some level of heuristic procedures to handle the problem. For the first approach, multiple sensors/actuators are simultaneously allocated. In the second approach, the sensor/actuators are placed in pairs, taking into account the influence of the spatial distribution of disturbances. Finally, the third approach provides a solution to the actuator location problem by incorporating considerations with respect to the preferred spatial regions within the flexible structure. All investigations are performed ranking the candidate positions based on a controllability index.

The proposed paper brings a discrete patch allocation which does not use a controllability index as usual in the revised papers. The metric used to highlight the suitability of the allocation is based directly in the Lyapunov functional, since it represents a balance between vibration suppression (remaining kinetic energy) and application of forces originated by the actuators (applied energy for vibration or displacement control). As it is assumed a discrete allocation and a predefined number of patches, the obtained results are not expected to be a global optimum, but it may be considered the optimal solution within those constraints.

2. Finite element formulation

2.1. The element GPL-T9 (generalised point and line compatibility triangular finite element with 9 DoF) for slender plates and shells

The incremental equilibrium equations using the finite element method (FEM) is given by [14,15]

$$[{}^tK]\{\Delta u\} = \{{}^{t+\Delta t}R\} - \{{}^tF\} \tag{1}$$

where $\{\Delta u\}$ is the vector containing the incremental nodal displacements and rotations, which is given by

$$\{\Delta u\} = \begin{Bmatrix} \{\Delta u_1\} \\ \{\Delta u_2\} \\ \{\Delta u_3\} \end{Bmatrix}; \quad \{\Delta u_i\} = \{\Delta u_{xi}\ \Delta u_{yi}\ \Delta w_i\ \Delta\theta_{xi}\ \Delta\theta_{yi}\ \Delta\theta_{zi}\}^T \tag{2}$$

with $(i = 1,2,3)$, ${}^{t+\Delta t}R$ is the vector of external nodal forces, ${}^tF$ is the internal force vector and $[{}^tK]$ is the stiffness matrix.

Taking into account coupling of membrane and bending effects for slender shells and plates, the following system of equations is obtained:

$$\begin{bmatrix} [K_m] & [K_{mb}] \\ [K_{bm}] & [K_b] \end{bmatrix} \begin{Bmatrix} \{\Delta u_m\} \\ \{\Delta u_b\} \end{Bmatrix} = \begin{Bmatrix} \{{}^{t+\Delta t}R_m\} - \{{}^tF_m\} \\ \{{}^{t+\Delta t}R_b\} - \{{}^tF_b\} \end{Bmatrix} \tag{3}$$

where the stiffness matrices due to effects of membrane $[K_m]$, membrane-bending coupling $[K_{mb}]$, $[K_{bm}]$ and bending $[K_b]$, are defined by

$$\begin{aligned} [K_m] &= \int_{{}^tA} [B_m]^T [D_m][B_m]\, {}^t dA; \\ [K_{mb}] &= \int_{{}^tA} [B_m]^T [D_{mb}][B_b]\, {}^t dA; \\ [K_{bm}] &= \int_{{}^tA} [B_b]^T [D_{bm}][B_m]\, {}^t dA = [K_{mb}]^T; \\ [K_b] &= \int_{{}^tA} [B_b]^T [D_b][B_b]\, {}^t dA. \end{aligned} \tag{4}$$

where $[B]$ and $[D]$ are the strain–displacement matrix and constitutive matrix, respectively, and m stands for membrane effects, b represents bending effects and mb indicates membrane-bending coupling effects.

These matrices are explicitly defined in the developed finite element code. The external nodal force vector referred to the membrane $\{{}^{t+\Delta t}R_m\}$ and bending effects $\{{}^{t+\Delta t}R_b\}$ are

$$\begin{aligned} \{{}^{t+\Delta t}R_m\} &= \int_{{}^tA} [H_m]^T \begin{Bmatrix} \{{}^{t+\Delta t}R_x\} \\ \{{}^{t+\Delta t}R_y\} \end{Bmatrix} {}^t dA; \\ \{{}^{t+\Delta t}R_b\} &= \int_{{}^tA} [H_b]^T \{{}^{t+\Delta t}R_z\}\, {}^t dA, \end{aligned} \tag{5}$$

where $[H]$ is the shape function matrix, $\{{}^{t+\Delta t}R_x\}$, $\{{}^{t+\Delta t}R_y\}$ and $\{{}^{t+\Delta t}R_z\}$ are the external nodal force vectors in the x, y and z direction, being the internal membrane and bending force vectors in the time t, $\{{}^tF_m\}$ and $\{{}^tF_b\}$, respectively, given by

$$\begin{aligned} \{{}^tF_m\} &= \int_{{}^tA} [B_m]^T \{{}^tN\}\, {}^t dA + \int_{{}^tA} [B_m]^T \{{}^tNM\}\, {}^t dA; \\ \{{}^tF_b\} &= \int_{{}^tA} [B_b]^T \{{}^tM\}\, {}^t dA + \int_{{}^tA} [B_b]^T \{{}^tMN\}\, {}^t dA. \end{aligned} \tag{6}$$

where $\{N\}$, $\{M\}$ and $\{NM\}$ or $\{MN\}$ are vectors of membrane force, bending per unit length and bending-membrane coupling, respectively.

For dynamic analysis, the equilibrium equation may be written as

$$[M]\{{}^{t+\Delta t}\ddot{u}\} + [C]\{{}^{t+\Delta t}\dot{u}\} + [{}^tK]\{\Delta u\} = \{\{{}^{t+\Delta t}R\} - \{{}^tF\}\} \tag{7}$$

The consistent mass matrix $[M]$ is given by

$$[M] = \sum_{k=1}^{n} h_k \rho_k \int_A [H]^T [H] dA \tag{8}$$

where n is the total number of composite layers, h_k is the thickness of the kth layer, ρ_k is the specific mass of the kth layer and the complete interpolation matrix $[H]$ is given by

$$[H] = \begin{bmatrix} L_i & 0 & H_{u\theta_i} & 0 & 0 & 0 \\ 0 & L_i & H_{v\theta_i} & 0 & 0 & 0 \\ 0 & 0 & 0 & H_i & H_{xi} & H_{yi} \end{bmatrix} (i = 1,2,3) \tag{9}$$

The damping matrix $[C]$ can be evaluated using the Rayleigh model

$$[C] = \alpha_R[M] + \beta_R[K] \tag{10}$$

where constants are determined with eigenvalues and damping ratios corresponding to two modes. More details may be found in Isoldi et al. [14].

2.2. Embedded piezoelectric material

If plies of piezoelectric material are added to the laminated composite material (as actuators and/or sensors), the electrical potential field must be included as an additional degree of freedom per node and per piezoelectric layer. The electric potential field increment is given by [16]

$${}^{t+\Delta t}_{0}\phi = {}^{t}_{0}\phi + \Delta\phi \tag{11}$$

Increment of the electric displacement vector is evaluated as follows:

$${}^{t+\Delta t}_{0}E_k t + \Delta t = -\frac{\partial({}^t_0\phi + \Delta\phi)}{\partial^0 x_k} = -\frac{\partial^t_0\phi}{\partial^0 x_k} - \frac{\partial\Delta\phi}{\partial^0 x_k} = {}^t_0E_1 - c_k \tag{12}$$

where ${}^t_0E_1$ is the gradient of the electric potential field. Then, the equations for the incremental electric displacement field is given by

$$\{\Delta E\} = \begin{Bmatrix} \Delta E_x \\ \Delta E_y \\ \Delta E_z \end{Bmatrix} = -\begin{Bmatrix} \Delta\phi_{,x} \\ \Delta\phi_{,y} \\ \Delta\phi_{,z} \end{Bmatrix} = -\begin{Bmatrix} \frac{\partial\Delta\phi}{\partial x} \\ \frac{\partial\Delta\phi}{\partial y} \\ \frac{\partial\Delta\phi}{\partial z} \end{Bmatrix} = -\begin{Bmatrix} 0 \\ 0 \\ \frac{\partial\Delta\phi}{\partial z} \end{Bmatrix} = -\begin{Bmatrix} 0 \\ 0 \\ \frac{\Delta\phi}{h_p} \end{Bmatrix} \tag{13}$$

When the finite element method is used, Eq. (13) is given by