Computers and Structures 128 (2013) 170–174

Contents lists available at ScienceDirect

Computers and Structures

journal homepage: www.elsevier.com/locate/compstruc

A lamination parameter-based strategy for solving an integer-continuous problem arising in composite optimization

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ARTICLE INFO

Article history:
Received 18 August 2012
Accepted 11 June 2013
Available online 13 September 2013

Keywords:
Optimization
Integer representation
Lamination parameters
Laminated composite

ABSTRACT

A bi-level optimization strategy for finding the optimal ply numbers and stacking sequence in composite structures has become one of the most popular techniques in recent years. When the optimization technique is based on the use of lamination parameters, the top level optimization has two subsets of design variables for each substructure (e.g., a panel in wing design): lamination parameters treated as continuous design variables, and three integers that define the number of plies of 0°, 90° and ±45° orientation. When a continuous optimizer is used at the top level, there is a need for an algorithm to find an integer representation of the obtained continuous number of plies that, ideally, does not alter the mechanical performance of a panel. The focus of this paper is on solving the top level optimization problem whereas the description of local level optimization problem that arranges the stacking sequence can be found in the authors' previous work. In order to determine the integer values of the ply numbers, two schemes based on the lamination parameter matching are introduced in this paper. The strategy is to use a binary code controlling the integer representation of ply numbers in order to obtain a discrete number of plies of each orientation per composite panel. An optimization problem is formulated where the objective function (to be minimized) defines how close the lamination parameter values and the panel thickness, obtained in the top level optimization, are to their values when integer ply numbers are considered. Such an optimization problem is solved by a permutation GA for each individual panel. A wing box benchmark problem is used to demonstrate the potential of these methods.

1. Introduction

Composite materials play an important role in aeronautical industry for tailoring the material's mechanical characteristics such as in-plane, flexural and bucking behaviour [1–4]. A bi-level optimization strategy for finding the optimal number of plies and stacking sequence in composite structures has been used in [5–11]. When the top level optimization is based on the use of lamination parameters, treated as continuous design variables, there is a need for an algorithm for an optimum integer representation of the continuous variable (thickness) so that the stacking sequence optimization can be performed to determine the detailed lay-up configuration of a composite laminated structure. Due to industrial requirements and manufacturing considerations, symmetric and balanced laminates with ply orientations of 0°, 90°, 45° and −45° are typically used resulting in a need to obtain three integer values of the number of 0°, 90° and ±45° plies per panel.

The use of lamination parameters is a convenient approach to representing the in-plane and flexural stiffness thus allowing for an efficient optimization of laminated composite structures. It was first used by Tsai et al. [12] and later applied to the buckling optimization of orthotropic laminated plates by Fukunaga and Hirano [13]. Miki [14] and Fukunaga [15] used lamination parameters for tailoring mechanical properties of laminated composites. In a laminated composite optimization problem, lamination parameters can be used as design variables instead of layer thicknesses and ply angles in order to avoid falling into local optima. Diaconu et al. [16] used a variational approach to determine feasible regions in the space of lamination parameters as constraints in the optimization problem. The soundness of the basic premise of looking for the nearest discrete solution in lamination parameter space has been called into question several times in the past, for example in paper [17]. It has been demonstrated that the optimum discrete solution is not necessarily the one nearest to the continuous solution in lamination parameter space. An alternative approach is to use the lamination parameters as intermediate variables for a surrogate model. In such an approach, the finite element analysis is replaced with a surrogate model of the structural response in terms of lamination parameters. Todoroki et al. [18,19] opted for a global response surface, while Herencia et al. [17] constructed a linear approximation of the design constraints around the

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http://dx.doi.org/10.1016/j.compstruc.2013.06.003

optimum continuous design to achieve the better laminates' stacking sequences in the lamination parameter design space.

In lamination parameter-based optimization, at the top level the out-of-plane lamination parameters and the numbers of plies of each fibre orientation (0°, 90° and ±45°) in each panel are treated as the design variables. The weight is the objective function to be minimized subject to the constraints on buckling, strength and lamination parameter feasibility. This is followed by the local level optimization for shuffling plies subject to the satisfaction of the lay-up rules, manufacturing and the mechanical performance preservation requirements (including blending constraints), see [9,10] for details.

The integer representation of ply numbers, that is the focus of this paper, can be viewed as a final stage of the top level optimization, if a continuous optimizer is used. In this stage, the continuous ply number values are converted into the integer ply numbers.

In this paper, two lamination parameter-based schemes are introduced for the integer representation of the ply number to interpret a continuous thickness given by the solution of the top level problem as integer numbers of plies of each orientation. In both schemes, procedure for the integer representation is separated from the local level optimization hence the lay-up rules, manufacturing requirements and mechanical performance preservation requirements (including ply blending) are not considered at the integer representation of ply numbers stage. The objective is to target the values of the lamination parameters obtained by the continuous optimizer in order to preserve the mechanical performance, also matching the overall thickness in each panel. The continuous representation of the number of plies is converted into the integer values for the number of plies of each orientation, which will be used as input data for the detailed stacking sequences of plies. The main difference between these two schemes is in whether matching of lamination parameters involves only the in-plane group or all lamination parameters including out-of-plane ones.

2. Lamination parameter-based method

The concept of lamination parameters was first introduced in [12]. The stiffness matrices A and D are governed by twelve lamination parameters and five material parameters. The A and D stiffness properties are derived from the classical laminate theory [20], which ignores transverse shear and normal stresses in the analysis of multilayered structures. Using this simple theory for composite analysis, the computational expense involved in the optimization of such structures can be significantly reduced. For more accurate analysis of multi-layered structures, the layer-wise analysis and zig-zag theories [21,22] could be used.

For orthotropic symmetric and balanced laminates, the number of independent lamination parameters can be reduced to eight. The elements of the membrane stiffness matrix A and the bending stiffness matrix D can be expressed as:

$$\begin{bmatrix} A_{11} \\ A_{22} \\ A_{12} \\ A_{66} \\ A_{16} \\ A_{26} \end{bmatrix} = h \begin{bmatrix} 1 & \xi_1^A & \xi_3^A & 0 & 0 \\ 1 & -\xi_1^A & \xi_3^A & 0 & 0 \\ 0 & 0 & -\xi_3^A & 1 & 0 \\ 0 & 0 & -\xi_3^A & 0 & 1 \\ 0 & \xi_2^A/2 & \xi_4^A & 0 & 0 \\ 0 & \xi_2^A/2 & -\xi_4^A & 0 & 0 \end{bmatrix} \begin{bmatrix} U_1 \\ U_2 \\ U_3 \\ U_4 \\ U_5 \end{bmatrix},$$

$$\begin{bmatrix} D_{11} \\ D_{22} \\ D_{12} \\ D_{66} \\ D_{16} \\ D_{26} \end{bmatrix} = \left(\frac{h^3}{12}\right) \begin{bmatrix} 1 & \xi_1^D & \xi_3^D & 0 & 0 \\ 1 & -\xi_1^D & \xi_3^D & 0 & 0 \\ 0 & 0 & -\xi_3^D & 1 & 0 \\ 0 & 0 & -\xi_3^D & 0 & 1 \\ 0 & \xi_2^D/2 & \xi_4^D & 0 & 0 \\ 0 & \xi_2^D/2 & -\xi_4^D & 0 & 0 \end{bmatrix} \begin{bmatrix} U_1 \\ U_2 \\ U_3 \\ U_4 \\ U_5 \end{bmatrix}. \tag{1}$$

where the lamination parameters are:

$\xi^A_{[1,2,3,4]} = \frac{1}{h}\int_{-h_i/2}^{h_i/2}[\cos 2\theta, \sin 2\theta, \cos 4\theta, \sin 4\theta]dz$ and $\xi^D_{[1,2,3,4]} = \frac{12}{h^3}\int_{-h_i/2}^{h_i/2}[\cos 2\theta, \sin 2\theta, \cos 4\theta, \sin 4\theta]z^2dz.$

This suggests that the use of lamination parameters as design variables in composite optimization can be very beneficial. It is known (see [1,16]) that the relationships between the out-of-plane lamination parameters can be expressed as:

$$\begin{aligned} &2\left(1+\xi_3^D\right)\left(\xi_2^D\right)^2 - 4\xi_1^D\xi_2^D\xi_4^D + \left(\xi_4^D\right)^2 \le \left(\xi_3^D - 2\left(\xi_1^D\right)^2 + 1\right)\left(1-\xi_3^D\right). \\ &\left(\xi_1^D\right)^2 + \left(\xi_2^D\right)^2 \le 1 \\ &2\left(\xi_1^D\right)^2 - 1 \le \xi_3^D \le 1. \end{aligned} \tag{2}$$

For the majority of aeronautical structures symmetric and balanced laminates with ply orientation of 0°, 90°, 45° and −45° are used. Thus, $\xi_4^D = 0$ and the first relationship in (2) can be rewritten as:

$$\left(\xi_2^D\right)^2 \le \frac{\left(\xi_3^D - 2\left(\xi_1^D\right)^2 + 1\right)\left(1-\xi_3^D\right)}{2\left(1+\xi_3^D\right)}. \tag{3}$$

Additional relationships between the in-plane and out-of-plane lamination parameters for symmetric laminates are available, see [17,23–26]. These expressions can be formulated as additional constraints for the top level optimization problem:

$$\begin{aligned}
&\left(\xi_i^A - 1\right)^4 - 4\left(\xi_i^D - 1\right)\left(\xi_i^A - 1\right) \le 0 \quad i = 1,2,3 \\
&\left(\xi_i^A + 1\right)^4 - 4\left(\xi_i^D + 1\right)\left(\xi_i^A + 1\right) \le 0 \quad i = 1,2,3 \\
&\left(2\xi_1^A - \xi_3^A - 1\right)^4 - 16\left(2\xi_1^D - \xi_3^D - 1\right)\left(2\xi_1^A - \xi_3^A - 1\right) \le 0 \\
&\left(2\xi_1^A + \xi_3^A + 1\right)^4 - 16\left(2\xi_1^D + \xi_3^D + 1\right)\left(2\xi_1^A + \xi_3^A + 1\right) \le 0 \\
&\left(2\xi_1^A - \xi_3^A + 3\right)^4 - 16\left(2\xi_1^D - \xi_3^D + 3\right)\left(2\xi_1^A - \xi_3^A + 3\right) \le 0 \\
&\left(2\xi_1^A + \xi_3^A - 3\right)^4 - 16\left(2\xi_1^D + \xi_3^D - 3\right)\left(2\xi_1^A + \xi_3^A - 3\right) \le 0 \\
&\left(2\xi_2^A - \xi_3^A + 1\right)^4 - 16\left(2\xi_2^D - \xi_3^D + 1\right)\left(2\xi_2^A - \xi_3^A + 1\right) \le 0 \\
&\left(2\xi_2^A + \xi_3^A - 1\right)^4 - 16\left(2\xi_2^D + \xi_3^D - 1\right)\left(2\xi_2^A + \xi_3^A - 1\right) \le 0 \\
&\left(2\xi_2^A - \xi_3^A - 3\right)^4 - 16\left(2\xi_2^D - \xi_3^D - 3\right)\left(2\xi_2^A - \xi_3^A - 3\right) \le 0 \\
&\left(2\xi_2^A + \xi_3^A + 3\right)^4 - 16\left(2\xi_2^D + \xi_3^D + 3\right)\left(2\xi_2^A + \xi_3^A + 3\right) \le 0 \\
&\left(\xi_1^A - \xi_2^A - 1\right)^4 - 4\left(\xi_1^D - \xi_2^D - 1\right)\left(\xi_1^A - \xi_2^A - 1\right) \le 0 \\
&\left(\xi_1^A + \xi_2^A + 1\right)^4 - 4\left(\xi_1^D + \xi_2^D + 1\right)\left(\xi_1^A + \xi_2^A + 1\right) \le 0 \\
&\left(\xi_1^A - \xi_2^A + 1\right)^4 - 4\left(\xi_1^D - \xi_2^D + 1\right)\left(\xi_1^A - \xi_2^A + 1\right) \le 0 \\
&\left(\xi_1^A + \xi_2^A - 1\right)^4 - 4\left(\xi_1^D + \xi_2^D - 1\right)\left(\xi_1^A + \xi_2^A - 1\right) \le 0.
\end{aligned} \tag{4}$$

In this paper, the lamination parameters are defined as:

$$\begin{aligned}
\xi_{0,i}^A &= \left(\frac{1}{h_i}\right)\int_{-h_i/2}^{h_i/2} 1dz = \frac{2\left(n_0^i + n_{90}^i + 2n_{45}^i\right)}{h_i} = 1 \\
\xi_{1,i}^A &= \left(\frac{1}{h_i}\right)\int_{-h_i/2}^{h_i/2} \cos 2\theta dz = \frac{2\left(n_0^i - n_{90}^i\right)}{h_i} = \frac{n_0^i - n_{90}^i}{n_0^i + n_{90}^i + 2n_{45}^i} \\
\xi_{2,i}^A &= \left(\frac{1}{h_i}\right)\int_{-h_i/2}^{h_i/2} \sin 2\theta dz = 0 \\
\xi_{3,i}^A &= \left(\frac{1}{h_i}\right)\int_{-h_i/2}^{h_i/2} \cos 4\theta dz = \frac{2\left(n_0^i + n_{90}^i - 2n_{45}^i\right)}{h_i} = \frac{n_0^i + n_{90}^i - 2n_{45}^i}{n_0^i + n_{90}^i + 2n_{45}^i} \\
\xi_{4,i}^A &= \left(\frac{1}{h_i}\right)\int_{-h_i/2}^{h_i/2} \sin 4\theta dz = 0.
\end{aligned} \tag{5}$$