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Nonlinear analysis methods based on the unstrained element length for determining initial shaping of suspension bridges under dead loads

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ABSTRACT

Two nonlinear analysis methods are newly presented for finding the initial shape of suspension bridges under dead load. For this, the extended tangential stiffness matrices of the nonlinear frame element as well as the elastic catenary cable element are derived by adding unstrained element lengths to the unknown. A G.TCUD (generalized target configuration under dead loads) method and an unstrained element length method are then proposed to get the minimized bending moment as well as to remove both lateral and axial deformations in the main girder and the tower. Finally the accuracy and effectiveness of the proposed schemes are demonstrated through two numerical examples.

1. Introduction

An initial configuration of conventional girder bridges can be easily determined by designer without the special analysis procedure because their displacements and deformations are so small under dead loads. Contrarily to girder bridges, it is well known that cable-supported bridges requires determination of the initial configuration satisfying the equilibrium condition between dead loads and internal member forces including cable tensions in the preliminary design stage because cable members display strongly geometric nonlinear behaviors as well as the configuration of a cable system cannot be defined in the stress-free state. This process determining the initial state of cable structures is referred to as "shape finding", "form finding" or "Initial shape or initial configuration" to express the target configuration at the initial equilibrium state under full dead loads. Furthermore as the span length of cable bridges increases extremely, it is very important to find an optimized solution of the initial configuration so that internal forces in cable-supported bridges under the full application of the dead loads are minimized and the deformed shape suits well to the initial target configuration. In other words, accurate and precise shape-finding analysis procedures should minimize the dead load-induced deformations by balancing the dead loads with pre-introduced internal member forces so that the final structure conforms to the designed configuration after the nonlinear analysis.

Shape-finding problems in cable-supported bridges are closely related to the determination of the initial cable force. The optimization technique [1], trial-and-error method [2], the initial force method [3–5], the TCUD method (target configuration under dead loads) [6], and the combined method [7] have been proposed up to present. The optimization techniques proposed by Takagi et al. [1] are based on a minimization technique to reduce the overall strain energy and the sum of squares of stresses accumulated in the girder, respectively. The trial-and-error method [2] utilizes an iterative procedure of updating cable tensions until the dead load deformations are confined within an allowable range. The initial force method [3,5] performs iterative nonlinear analysis of the structure until the initial forces introduced in the frame and cable elements converge to the unknown target values and the structural system comes to satisfy self-equilibrium state between dead loads and internal member forces. However it should be pointed out that the initial force methods can lead to somewhat complex numerical algorithm because bending moments as well as axial forces are included in initial forces and the horizontal tension component of the main cable is iteratively updated.

On the other hand, realizing that the initial shaping problem of cable-supported bridges is a kind of inverse problem, Kim and Lee [6] proposed a TCUD method that implies the target configuration under dead loads for cable-supported bridges. In this scheme, the successive substitution procedure is not required in shape-finding analysis by considering the unstrained length of each cable as unknown parameter in formulating stiffness matrix. Since the number of unknown parameters increases to the amount of the

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number of cable members, a corresponding number of geometric constraints can be added to the general boundary conditions in solving the simultaneous equilibrium equations of the system. This method has provided analytical formulations to prevent undesired deformation of active degrees of freedom as many as the number of cable members. However, the shortenings of stiffening truss and main tower due to axial force are not avoidable due to the limitation in the number of constraints provided when a self-anchored suspension bridge is considered.

To overcome these weak points, Kim and Kim [7] recently presented an effective method to be able to eliminate those axial deformations as well as to preserve merits of the TCUD method in which the TCUD method and the initial force method with successive substitution were efficiently combined based on an elastic catenary cable element. This scheme clearly improved the TCUD method but can lead to somewhat complex computer programming due to the iteration process with successive substitution. In addition, it should be pointed out that the nonlinear frame elements adopted by Kim et al. [5] and Kim and Lee [6] are somehow approximate because it is assumed that frame elements are not curved but straight in the deformed state and the internal forces within frame elements are evaluated using an accumulation method where the incremental internal forces are calculated using incremental displacements and the total internal forces are then updated at each iterative step.

On the other hand, the equilibrium state solution obtained by the initial shaping analysis should be not only the starting point of nonlinear static/dynamic load analyses or forward/backward construction step analysis but also provide valuable information on member fabrication or geometry control during erection. From this point of view, the TCUD methods are believed to give an ideal solution including unstrained element lengths for the initial configuration but are not always effective in performing nonlinear analyses for various load combination or construction stage analysis because of the additional geometric constraints.

Based on this research background, this paper intends to propose relatively clear and improved nonlinear analysis methods that can provide an optimized solution for the initial state of for long-span suspension bridges based on unstrained element lengths of both beam-column members and cables. The main issues and contributions can be summarized as follows:

(1) An incremental equilibrium equation is summarized for an elastic catenary cable element in which the unstrained length of each cable element is considered as the unknown variables. In addition, the generalized tangential stiffness matrix of a consistent frame element based on the co-rotational formulation is newly derived including the unstrained length as the unknown.
(2) An analytical method determining initial tensions of cable elements as well as initial shapes of the cable system is next explained which serves as trial values in the first iteration process of the TCUD methods.
(3) The *generalized TCUD algorithm* is then proposed by including the unstrained lengths of both the frame element and the cable element as unknowns and deriving the extended tangential stiffness matrix of the bridge structure. Resultantly additional geometric constraints equal to the total number of finite elements are imposed to prevent axial as well as lateral displacements of the main towers and girders.
(4) In addition to this, the *unstrained element length method* (*UELM*) is newly presented based the conventional Newton iteration method in which contrarily to TCUD methods, the *symmetric* tangential stiffness matrix is used without modification and the unstrained lengths of both the frame element and the cable element are kept constant in the iteration process.
(5) Finally the accuracy and effectiveness of the proposed algorithms are demonstrated through the numerical application for two suspension bridges.

2. Cable and frame elements considering their unstrained length as an unknown

Firstly the nonlinear formulation of the elastic catenary cable element is briefly summarized in this section and the extended tangential stiffness matrix of a nonlinear frame element is then derived by considering the unstrained length as the unknown.

2.1. An elastic catenary cable element

An elastic catenary cable element (Jayaraman and Knudson [8]) has been derived from the exact solution (Irvine [9]) of the elastic catenary cable equation, deformed due to its self-weight. Afterward the improved elastic catenary cable elements (Kwan [10], Andreu et al. [11] and Vu et al. [12]) have been proposed for nonlinear analysis of cable structures. It can be formulated in three-dimensional coordinates but only two-dimensional formulation is described in this study.

Consider a cable element suspended between points $i(0,0)$ and $j(L_x, L_y)$ as shown in Fig. 1. The relative distances between the two nodes along the global x, y axis, which are denoted by L_x and L_y, respectively in Fig. 1, can be expressed as a function of the global nodal force F_1 and F_2 at the node i as

$$L_x = -\frac{F_1 L_o}{EA_o} + \frac{F_1}{w}\left\{\sinh^{-1}\left(\frac{F_2 - wL_o}{F_1}\right) - \sinh^{-1}\left(\frac{F_2}{F_1}\right)\right\} \tag{1a}$$

$$L_y = -\frac{F_2 L_o}{EA_o} + \frac{wL_o^2}{2EA_o} + \frac{1}{w}(T_q - T_p) \tag{1b}$$

where $T_p = \sqrt{F_1^2 + F_2^2}$ and $T_q = \sqrt{F_1^2 + (F_2 - wL_o)^2}$ and EA_o = the axial rigidity; L_0 = the unstrained length; w = the self-weight per unit length of the cable. In case of hanger cables inclined slightly from the vertical state in the iteration process, the horizontal component F_1 in Eq. (1a) can be nearly zero which leads to a singularity problem. To avoid this difficulty, Eq. (1a) can be written by Taylor series expansion as follows;

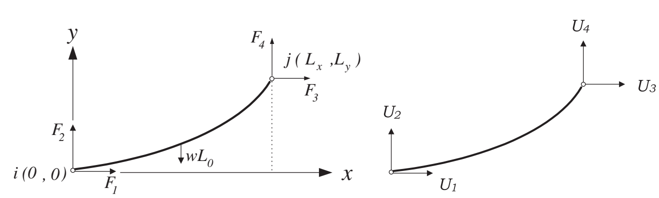
$$L_x = -\frac{F_1 L_o}{EA_o} - L_o + \frac{1}{w}(T_p - T_q) \tag{2}$$

Now partial differentiation of both sides of Eq. (1) yields the following incremental relationships between the relative displacements and nodal forces in matrix notation.

$$\begin{Bmatrix} \Delta L_x \\ \Delta L_y \end{Bmatrix} = \begin{bmatrix} \partial L_x/\partial F_1 & \partial L_x/\partial F_2 \\ \partial L_y/\partial F_1 & \partial L_y/\partial F_2 \end{bmatrix} \begin{Bmatrix} \Delta F_1 \\ \Delta F_2 \end{Bmatrix} + \begin{bmatrix} \partial L_x/\partial L_o \\ \partial L_y/\partial L_o \end{bmatrix} \Delta L_0 \tag{3a}$$

$$\Delta L_x = \Delta U_3 - \Delta U_1 \quad \text{and} \quad \Delta L_y = \Delta U_4 - \Delta U_2 \tag{3b, c}$$

where the detailed form of coefficients in Eq. (3a) can be referred to Kim and Kim [7] and the results for Eq. (2) and (1b) can be similarly derived.



(a) Nodal force vectors (b) Nodal displacement vectors

Fig. 1. Nodal force and displacement vectors of an elastic catenary cable element.