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Temporal finite elements for target control dynamics of mechanisms

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Abstract

This paper discusses a temporal finite element description in the analysis of dynamics of mechanical systems, with a special emphasis on problems where target control is desired. This is defined as a situation where forces are sought for the movement of a structure from an initial to one or more specified target states. The primary applications lie in robotics and in bio-mechanical forward simulations of musculoskeletal systems. A temporal discretization of the movement and forces is introduced. By interpolating simultaneously displacements and velocities in the spatial discrete degrees of freedom, a collocation over the time interval can be used to decide the necessary system. The needed control can be optimized for chosen criteria on the integrated force components. The temporal interpolation of control forces and discrete displacements introduces a degree of continuity in the obtained results. The viewpoint allows variation of many aspects of problem formulation, and leads to efficient solutions for systems of high complexity but moderate size.

Keywords: Musculoskeletal system; Biomechanics; Dynamics; Target states; Mechanisms; Finite element approximation

1. Introduction

This paper discusses a temporal finite element approximation in the analysis of dynamics of mechanical systems, with a special emphasis on problems where target control is desired. This is defined as a situation where forces are to be introduced for the movement of a structure from an initial to one or more target states, but where the behavior between these states is arbitrary. The primary applications lie in robotics and in bio-mechanical simulations of musculoskeletal systems, where forces are needed to change the configuration, and these forces are preferably kept minimal.

The problems are seen in a discrete form, which can correspond to the basic mechanical formulation, or to a spatially discretized form of a continuous problem. It is thereby assumed that the kinematics of a considered system is described by a limited number of displacement coordinates.

For mechanical systems with only pre-defined external forces, the temporal finite element description can be seen as an alternative to other methods for the direct integration of the dynamic equations. The problem is here defined by the load distribution and the initial values of all displacements and their time differentials. Linearity in the governing equation is not necessary

A large number of direct integration methods exist in the literature, [1–4]. One class consists of general time-stepping algorithms for first-order initial value problems. They need a rewriting of the dynamic second order equilibrium equations into a set of first order state equations:

$$\boldsymbol{z} = \begin{pmatrix} \boldsymbol{v} \\ \boldsymbol{d} \end{pmatrix}; \quad \dot{\boldsymbol{z}} = \begin{pmatrix} \dot{\boldsymbol{v}} \\ \dot{\boldsymbol{d}} \end{pmatrix} = \begin{pmatrix} \boldsymbol{a} \\ \boldsymbol{v} \end{pmatrix} = \dot{\boldsymbol{z}}(\boldsymbol{z}) \tag{1}$$

where $\boldsymbol{d}$, $\boldsymbol{v}$, $\boldsymbol{a}$ are displacements, velocities and accelerations. The top part of the time differential contains the structural dynamics formulation, whereas the lower part is trivial. The first order system of doubled size can be integrated with any of the general methods.

A second class of methods recognizes the relation between velocity and acceleration, and is more adapted

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for the second order problem, e.g., the Newmark or Hilber–Hughes–Taylor methods. Several versions exist, but here only the 'linear acceleration' Newmark method described as $\gamma = \frac{1}{2}$, $\beta = \frac{1}{6}$ is considered, [2].

The proposed methodology is primarily aimed at target control problems, where a number of target states are demanded to be fulfilled at certain time stations. In addition to known external forces, unknown control forces are introduced to give the desired movement. The alternative would have been to develop a shooting method, using an iterative variation of the control force values, and adopting any of the above-mentioned methods for time integration of an evolution problem. The systematic variation of control forces would, if such a method were developed, lead to a non-linear set of equations, concerned with the target state displacements. Obviously, such a method would be computationally expensive, and often unreliable for complex and large problems. The proposed method replaces this with a larger, but one-stage, procedure.

The target control problem allows two viewpoints in the numerical simulation. The difference lies in the chosen discretization for the control forces, and in particular the number of unknown force components introduced in the problem statement. With a minimal number of such components, unique solutions are sought, whereas a higher number gives a continuous set of possible solutions. The minimal number is related to the number of boundary conditions on the displacements and velocities. In the non-unique case, an optimal control force history can be sought, for instance demanding that some measure, "cost", of the introduced forces should be minimal. In the present work, the two cases are denoted fixed control and optimal control problems, respectively. The mathematical forms are quite different, but they can be dealt with in a common algorithm.

The optimality of human movement is a much debated issue. In the present work, it is assumed that the optimality is measured by some cost function for the needed control forces; the specific choice can be easily replaced in the algorithm. Another interesting possibility is to emphasize the physiological demand for smooth motion patterns, by basing the optimality on jerk measures. Variations of the minimum-jerk model formulated by Flash and Hogan, [5], have been used both in the diagnosis of pathology, [6], and as a tool for evaluating the kinematics of both human and robotic or prosthetic movement, [7–9]. Whether any criterion is valid for all kinds of movements and in all situations is not discussed here, cf. [10].

The paper gives the basic equations for the temporal finite element formulation of three classes of dynamic problems, and develops a common algorithm. The starting point is a dynamic equilibrium formulation for the mechanical system. The system is subjected to prescribed external force variations, but also initial conditions, and possibly target states during a considered time interval. The time variation of the displacement state is here introduced by a Hermitian finite element form, where each coordinate is represented by its value and its time differential at a set of discrete time stations. All displacement variables are thereby represented as piecewise cubic over time. The solution gives displacement values and values for the a priori unknown control forces.

The methodology allows any complexity of the dynamic equations. The treatment of a specific problem is thereby simplified, demanding just a problem-specific function, delivering some well-defined quantities, as functions of the current solution.

Performed tests indicate that the developed viewpoint and algorithm can be efficient in the study of complex, but primarily moderate size problems, with an improved continuity in the description of motion, and a good stability in the dynamic solution. Some relevant tests are discussed in the paper, as are the necessary improvements to the formulation needed to allow the treatment of many classes of interesting bio-mechanical simulations.

2. Basic formulation

2.1. Time instance formulation

The mechanical problem was seen as described by a discretized dynamic equilibrium equation:

$$[\mathbf{M}]\boldsymbol{a}^t + \boldsymbol{f}(\boldsymbol{u}^t, \boldsymbol{v}^t) - \boldsymbol{p}^t - [\mathbf{F}_c]\boldsymbol{c}^t = \mathbf{0} \tag{2}$$

where the vectors $\boldsymbol{u}^t$, $\boldsymbol{v}^t$, $\boldsymbol{a}^t$ and $\boldsymbol{p}^t$ denote displacements, velocities, accelerations and displacement-independent external forces in a set of N_d degrees of freedom, at a time instance, t. In the equation, the matrix $\mathbf{M}$ is the mass matrix for the discrete system. The vector $\boldsymbol{c}^t$ contains a set of N_c control forces, whose attachments to the degrees of freedom are described by the action description matrix $\mathbf{F}_c$, of size N_d-by-N_c. Forces related to current displacements and velocities at the time instance are formulated allowing general expressions, the simplest being a linear relation:

$$\boldsymbol{f}(\boldsymbol{u}^t, \boldsymbol{v}^t) \equiv [\mathbf{K}]\boldsymbol{u}^t + [\mathbf{D}]\boldsymbol{v}^t \tag{3}$$

with linear stiffness and damping matrices, and with no displacement dependent external forces. The external forces in $\boldsymbol{p}$ are prescribed for the studied time interval. In the general case, the equilibrium equation is obtained through the Lagrange energy equations, [11].

The solution to the problem was seen as the variation of the displacements:

$$\boldsymbol{u}^t = [u_1(t), u_2(t), \ldots, u_{N_d}(t)]^{\mathrm{T}} \tag{4}$$

for $t \geqslant 0$. If present, the solution also contains the time variation of the control forces:

$$\boldsymbol{c}^t = [c_1(t), c_2(t), \ldots, c_{N_c}(t)]^{\mathrm{T}} \tag{5}$$