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Nonlinear inelastic uniform torsion of composite bars by BEM

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ABSTRACT

In this paper the elastic–plastic uniform torsion analysis of composite cylindrical bars of arbitrary cross-section consisting of materials in contact, each of which can surround a finite number of inclusions, taking into account the effect of geometric nonlinearity is presented employing the boundary element method. The stress–strain relationships for the materials are assumed to be elastic–plastic–strain hardening. The incremental torque–rotation relationship is computed based on the finite displacement (finite rotation) theory, that is the transverse displacement components are expressed so as to be valid for large rotations and the longitudinal normal strain includes the second-order geometric nonlinear term often described as the "Wagner strain". The proposed formulation does not stand on the assumption of a thin-walled structure and therefore the cross-section's torsional rigidity is evaluated exactly without using the so-called Saint Venant's torsional constant. The torsional rigidity of the cross-section is evaluated directly employing the primary warping function of the cross-section depending on both its shape and the progress of the plastic region. A boundary value problem with respect to the aforementioned function is formulated and solved employing a BEM approach. The influence of the second Piola–Kirchhoff normal stress component to the plastic/elastic moment ratio in uniform inelastic torsion is demonstrated.

1. Introduction

When a bar is subjected to uniform torque arising from two concentrated torsional moments at its ends while the warping of the cross-section is not restrained, the angle of twist per unit length remains constant along its axis and the bar is under uniform torsion. Though uniform torsion rarely occurs in practice due to changes in the torque or restraints against twisting and warping, uniform torque forms one component of the total nonuniform torque, the other being the warping torque.

Designs based on elastic analysis are likely to be extremely conservative not only due to the significant difference between first yield in a cross-section and full plasticity but also due to the unaccounted for yet significant reserves of strength that are not mobilized in redundant members until after inelastic redistribution takes place. Besides, since thin-walled open sections have low torsional stiffness, the torsional deformations can be of such magnitudes that it is not adequate to treat the angles of cross-section rotation as small. Thus, both material inelasticity and geometric nonlinearity are important for investigating the ultimate strengths of beams that fail by torsion. Moreover, in recent years composite structural elements consisting of a relatively weak matrix reinforced by stronger inclusions or of different materials in contact are of increasing technological importance in engineering. Composite structures can produce very elegant solutions to complex structural engineering challenges, while composite beams or columns offer many significant advantages, such as high load capacity with small cross-section and economic material use, simple connection to other members as for steel construction, good fire resistance etc. Steel beams or columns totally encased in concrete are most common examples.

Several researchers have dealt with the elastic–plastic uniform torsional behavior of homogeneous beams with the pioneering work of Nadai [1] who developed the sand-heap analogy for the full plastic torque of solid sections. Christopherson [2] obtained an elastic–plastic solution for an I-section, later Nadai [3] used the rooftop membrane analogy for the elastic–plastic solution of various cross-sections, Sokolovsky [4] developed an elastic–plastic solution for an oval section, Smith and Siderbottom [5] derived an elastic–plastic solution for prismatic bars of rectangular sections and Billinghurst et al. [6] used the mitre method to obtain elastic–plastic solutions for various cross-sections.

According to the nonlinearity induced by finite twist rotation angles, Ashwell [7] and Gregory [8] studied both theoretically and experimentally the elastic nonlinear behaviour of twisted cantilevers of different cross-sections under uniform torsion conditions, while Tso and Ghobarah [9] presented a study of the nonlinear nonuniform elastic torsion of thin-walled open sections.

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Recently, numerical methods have also been used to investigate the elastic–plastic uniform and nonuniform torsional behaviour of beams. Yamada et al. [10], Johnson [11], and Itani [12] studied elastic–plastic uniform torsion. Baba and Kajita [13] used a two-node, four-degree-of-freedom beam element for the uniform torsion analysis and a four-node, 12-degree-of-freedom rectangular section element for the warping analysis of the section. Bathe and Chaudhary [14] used warping displacement functions for beams of rectangular cross-section in the formulation of a two-node Hermitian-based beam, and in the formulation of a variable number of nodes isoparametric beam for the linear and nonlinear analysis of torsion. Bathe and Wiener [15] employed a Hermitian beam element and a nine-node shell element for the elastic–plastic nonuniform torsion of I-beams. Gellin et al. [16] presented a strip finite-element model for the analysis of the nonlinear material behavior of thin-walled members in nonuniform torsion. May and Al-Shaarbaf [17] used a standard three-dimensional 20-node isoparametric quadratic brick element in the elastic–plastic analysis of uniform and nonuniform torsion of members subjected to pure and warping torsion. Chen and Trahair [18] using the mitre model to describe the shear strain distribution over the cross-section and Pi and Trahair [19–21] using the principle of virtual work developed a finite-element model for the inelastic analysis, plastic design and plastic collapse of nonuniform torsion of I-section thin walled beams. Finally, Wagner and Gruttmann [22] developed associated isoparametric finite elements based on variational formulation to analyze the uniform elastic–plastic torsion problem of prismatic bars of arbitrary cross-section. All of the aforementioned research efforts concern the torsion problem of homogeneous bars, while composite bars have not yet been examined. Also, to the authors' knowledge the boundary element method has not yet been used for the numerical analysis of the aforementioned problems.

In this paper the elastic–plastic uniform torsion analysis of composite cylindrical bars of arbitrary cross-section consisting of materials in contact, each of which can surround a finite number of inclusions, taking into account the effect of geometric nonlinearity is presented employing the boundary element method. The stress–strain relationships for the materials are assumed to be elastic–plastic–strain hardening. The incremental torque–rotation relationship is computed based on the finite displacement (finite rotation) theory, that is the transverse displacement components are expressed so as to be valid for large rotations and the longitudinal normal strain includes the second-order geometric nonlinear term often described as the "Wagner strain" [20]. The torsional rigidity of the cross-section is evaluated directly employing the primary warping function of the cross-section [23] depending on both its shape and the progress of the plastic region. A boundary value problem with respect to the aforementioned function is formulated and solved employing a BEM approach. The proposed formulation procedure is based on the assumption of no local or lateral torsional buckling or distortion and includes the following essential features and novel aspects compared with previous ones:

(i) Large deflections and rotations are taken into account, that is the strain–displacement relationships contain higher order displacement terms.
(ii) For the first time in the literature, the influence of the second Piola–Kirchhoff normal stress component on the plastic/elastic moment ratio in uniform inelastic torsion is demonstrated.
(iii) For each one of the materials of the cross-section, material inelasticity is taken into account, that is the elastic–plastic incremental stress–strain relationship is derived from the von Mises yield criterion, a strain flow rule and a strain hardening rule. Integrations of stress resultants for every iterative step and restoration of equilibrium for every converged incremental step are performed numerically using a set of monitoring stations distributed over the area of the cross-section.
(iv) The present formulation is applicable to bars of arbitrary composite cross-section, while the case of a homogeneous cross-section can be treated as a special one.
(v) The presented formulation does not stand on the assumption of a thin-walled structure and therefore the cross-section's torsional rigidity is evaluated exactly without using the so-called Saint Venant's torsional constant.
(vi) The boundary conditions at the interfaces between different material regions have been taken into account.
(vii) The proposed method can be efficiently applied to composite beams of thin or thick walled cross-section and to laminated composite beams. Previous formulations concerning composite beams of thin walled cross-sections or laminated cross-sections are analyzing these beams using the '*refined models*'. However, these models analyze the beam with respect to cross-section mid lines ignoring the warping along the thickness of the walls. Moreover, they do not satisfy the continuity conditions of transverse shear stress at layer interfaces and assume that the transverse shear stress along the thickness coordinate remains constant, leading to the fact that kinematic or static assumptions cannot be always valid [24–26].

Numerical results are presented to illustrate the method and demonstrate its efficiency and accuracy. The contribution of the normal stresses is investigated by numerical examples with great practical interest.

2. Statement of the problem

Consider a bar of length l with an arbitrarily shaped composite cross-section, consisting of materials in contact, each of which can

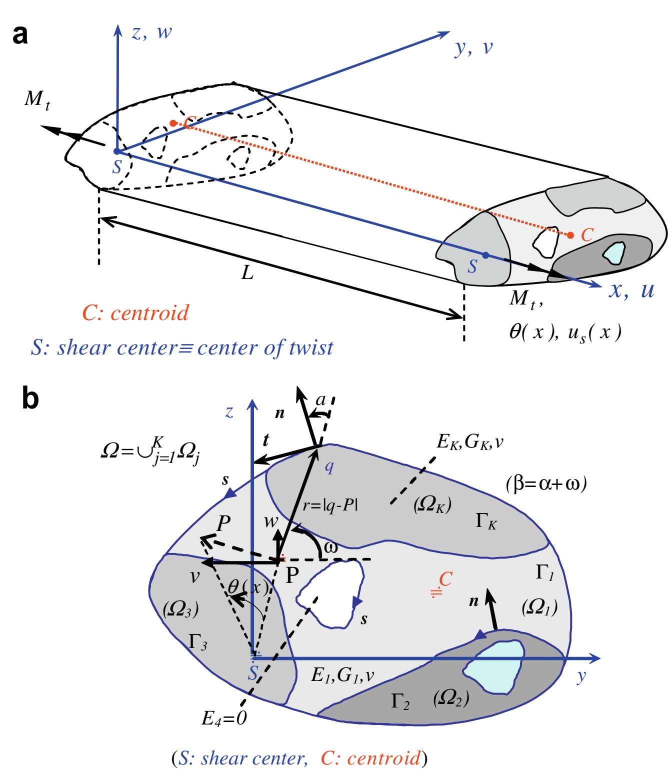

Fig. 1. Prismatic bar subjected to a twisting moment (a) with a composite cross-section of arbitrary shape occupying the two dimensional region Ω (b).