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Two-Stage Stochastic Mixed-Integer Linear Programming: The Conditional Scenario Approach

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Abstract

In this paper we consider the two-stage stochastic mixed-integer linear programming problem with recourse, which we call the RP problem. A common way to approximate the RP problem, which is usually formulated in terms of *scenarios*, is to formulate the so-called Expected Value (EV) problem, which only considers the expectation of the random parameters of the RP problem. In this paper we introduce the *Conditional Scenario* (CS) problem which represents a midpoint between the RP and the EV problems regarding computational tractability and ability to deal with uncertainty. In the theoretical section we have analyzed some useful bounds related to the RP, EV and CS problems. In the numerical example here presented, the CS problem has outperformed both the EV problem in terms of solution quality, and the RP problem with the same number of scenarios as in the CS problem, in terms of solution time.

Keywords: Stochastic mixed-integer linear programming, conditional expectation, scenario, conditional scenario.

1 Introduction

The relevance for managerial purposes, properties, solution methods and applications of two-stage stochastic mixed-integer linear programming can be found in surveys such as [35] and in books such as [10]. Among the applications one finds: the facility location problem with Bernoulli demands [2], scheduling of a multiproduct batch plant with uncertain demand [15], strategic production planning under uncertainty [4], thermal power system expansion [3] and employee scheduling in retail outlets with uncertain demand [30].

To address stochastic optimization problems different approaches can be used: robust optimization [7], chance constraint optimization [13, 32], sampling based methods [21] and scenario based optimization [10, 25], among others. In this paper we focus on the last approach. The RP problem here considered corresponds to a *two-stage stochastic mixed-integer linear* optimization problem with *recourse* and *risk neutral*. Several variants of this RP problem have been proposed in the literature: multi-stage versions [8, 28], risk aversion versions [37], non-linear versions [1], etc.

The two-stage stochastic mixed-integer linear programming problem formulated in terms of a continuous random vector which accounts for all the uncertain parameters of the problem is, in general, numerically intractable. To address this difficulty one can approximate the original random vector by a random vector with a finite number of realizations (the *scenario tree*). Thus, in the first step of scenario

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