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Integrated framework for robustness analysis using ratio-based efficiency model with application to evaluation of Polish airports ☆

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A B S T R A C T

We consider a problem of evaluating efficiency of Decision Making Units (DMUs) based on their deterministic performance on multiple consumed inputs and multiple produced outputs. We apply a ratio-based efficiency measure, and account for the Decision Maker's preference information representable with linear constraints involving input/output weights. We analyze the set of all feasible weights to answer various robustness concerns by deriving: (1) extreme efficiency scores and (2) extreme efficiency ranks for each DMU, (3) possible and necessary efficiency preference relations for pairs of DMUs, (4) efficiency distribution, (5) efficiency rank acceptability indices, and (6) pairwise efficiency outranking indices. The proposed hybrid approach combines and extends previous results from Ratio-based Efficiency Analysis and the SMAA-D method. The practical managerial implications are derived from the complementary character of accounted perspectives on DMUs' efficiencies. We present an innovative open-source software implementing an integrated framework for robustness analysis using a ratio-based efficiency model on the *diviz* platform. The proposed approach is applied to a real-world problem of evaluating efficiency of Polish airports. We consider four inputs related to the capacities of a terminal, runways, and an apron, and to the airport's catchment area, and two outputs concerning passenger traffic and number of aircraft movements. We present how the results can be affected by integrating the weight constraints and eliminating outlier DMUs.

1. Introduction

The framework of Data Envelopment Analysis (DEA) offers a variety of methods for evaluating the relative efficiency of Decision Making Units (DMUs) which consume multiple inputs and produce multiple outputs [18,39,38]. Conceptually, efficiency is the ratio between virtual output and virtual input, i.e., respectively, outputs or inputs aggregated using some weights assigned to these factors [14]. Typically, DEA methods have been used to classify the DMUs into efficient and inefficient ones. By definition, the former ones have an efficiency score equal to one, whereas for the latter ones this measure is less than one. For the inefficient DMUs, such scores convey information on how close to being efficient they are. Analysis of these measures may lead to formulating the corrective actions, revealing an excess use of some inputs or shortfalls in the production of outputs, as well as to indicating a reference set of some comparable DMUs.

1.1. *Critical view on the traditional methods of data envelopment analysis*

Although DEA has proven its usefulness when applied to a variety of real-world problems (see, e.g., [23,18,40]), some criticism has been leveled against its discriminative power and the way the efficiency scores are computed. Firstly, the efficiency measures for each DMU are derived from the analysis of the input/output weights which are the most favorable to it. However, a weight vector for which a DMU attains its maximal efficiency is not unique [36]. Thus, choosing among them is arbitrary to a large extent. Secondly, the underlying Linear Programming (LP) techniques require some normalization of weights for each DMU individually. This implies that scaling affects the optimal weights and a meaningful comparison of these weights across different DMUs is difficult. Thirdly, the efficiency measures fail to reflect how the efficiencies of DMUs compare to each other for other feasible weight vectors [53]. In fact, only extremely small share of feasible weights is taken into account in the analysis, while others are neglected despite being equally desirable. Fourth, DEA measures efficiency relative to the efficient frontier. This requires some assumptions about possible returns to scale (e.g., constant or variable). These may be, however, difficult to formulate or justify.

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Further, we may sometimes prefer a DMU judged as inefficient, which is dominated only by some convex combination of other DMUs, but not by any existing DMU [36]. Moreover, an efficiency frontier and, thus, the efficiency scores, vastly depend on the DMUs under consideration [74,58]. The outcomes of DEA may be very sensitive even to the inclusion or removal of a single DMU. In the same spirit, the outcomes of DEA can be interpreted only when the number of DMUs is large enough in comparison with the number of inputs and outputs. Finally, while DEA is useful for indicating which DMUs are efficient, it does not discriminate between them. In some real-world situations, the share of efficient DMUs may be very large, and we may wish to identify among them a small subset of the most distinguishing ones.

Several techniques have been proposed in the literature to address these drawbacks. In particular, preference information on the relative comparisons of inputs and/or outputs may be used to reduce the space of feasible weight vectors [63,50], and, thus, the conclusiveness of efficiency scores. Further, the cross-efficiency methods exploit the space of feasible weights to derive for each DMU an average efficiency obtained from the analysis of weights for which other DMU's efficiency is maximal [59,22]. Moreover, the super-efficiency discriminates the efficient DMUs by indicating for each of them how much more efficient it can be relative to the remaining ones [2,75]. Although following the right direction, these approaches do not address all aforementioned concerns comprehensively. Doing so, requires incorporation of robustness analysis into the DEA framework.

1.2. Existing approaches for robustness analysis in data envelopment analysis

Robustness analysis accounts for the uncertainties which can be observed in the real-world decision problems [33]. A conclusion is considered to be robust if it is true for all or for the most plausible combinations of parameter values [52,67]. As noted in [20], this type of analysis provides information that may allow the users to avoid answering questions they find too demanding. It may also guide them in revising or enriching the provided preference information, progressively constraining the space of admissible values for the parameters of employed model. In the context of DEA, robustness concern refers to the relative efficiencies of DMUs for all feasible input and output weights or their representative sample. Advances in this regard, that we build on in this paper, have been presented in [53] and [36].

On one hand, [53] consider the whole set of weights that are compatible with the preference information concerning input/output variables. The so-called Ratio-based Efficiency Analysis (REA) does not make any assumptions in terms of the production possibilities beyond the set of DMUs that are under comparison. To materialize the relations between the DMUs' efficiencies, the method exhibits three kinds of results derived from the analysis of the whole set of feasible weights: efficiency bounds exposing the greatest and the least relative efficiencies of a DMU compared to a subset of other DMUs, dominance relation indicating for a pair of DMUs if one of them dominates the other in pairwise efficiency comparison, and ranking intervals indicating the range of efficiency ranks that are attained by a DMU. All these results are derived from comparing DMUs' efficiencies pairwise rather than measuring their distance from an efficient frontier as in the traditional DEA models. As a result, these outcomes are interpretable even if the set of DMU is relatively small, being at the same time less sensitive to the inclusion of DMU whose input/output values are distant from the performances of other units.

On the other hand, [36] apply simulation to provide stochastic indices which characterize the possible outcomes of a decision problem. In Stochastic Multicriteria Acceptability Analysis for Data Envelopment Analysis (SMAA-D), it is possible to handle imprecision and uncertainty regarding the input/output weights and performances of DMUs. The method computes rank acceptability indices which measure the variety of model variables that grant each DMU any rank from the best to the worst. In particular, the best (most acceptable) DMUs are those with high acceptabilities for the first rank. When compared with the basic DEA models, the stochastic measures originally provided in SMAA-D have been found useful for making the efficient DMUs more comparable [36].

1.3. Aim of the paper

The aim of this paper is fourth-fold. Firstly, from a methodological point of view, we extend the range of outcomes considered in REA and SMAA-D. With respect to the robustness analysis, we show how to determine the least efficiency measure for each DMU, i.e., what is the lower bound of the efficiency range when the whole set of DMUs (including the DMU under consideration) is analyzed. When considering stability of the efficiency comparison for pairs of DMUs, we propose to consider the necessary and possible efficiency preference relations instead of the dominance relation. The necessary relation needs to be confirmed by all feasible weight vectors, while the possible one has to be supported by at least one feasible weight vector. We show that taking into account these results is more beneficial than analyzing the dominance relation because of their interpretability and intuitive convergence with the growth of the preference information for input/output variables.

When it comes to SMAA-D, we significantly enrich the range of stochastic indices that can be derived from the representative sample of weight vectors so that they additionally capture the efficiency scores and pairwise efficiency relations. In particular, we analyze the extreme observed efficiencies, the distribution of efficiency measures, and pairwise efficiency outranking (winning) indices indicating the probability that one DMU has an efficiency at least as good (better) than the other. In this way, we provide both exact and stochastic outcomes reflecting three different perspectives on DMUs' efficiency: scores, pairwise preference relations, and attained ranks.

Secondly, we clearly demonstrate the benefits of considering together the outcomes of thus revised REA and SMAA-D. On one hand, with the necessary, possible, and extreme outcomes of the revisited REA, we can analyze what happens for all, some, the most and the least advantageous model parameters. However, the difference between extreme ranks and efficiencies may often be very large, and in practical decision analysis the information on the sole possibility of attaining a particular rank or an efficiency in a given subinterval may be insufficient. Similarly, REA leaves incomparable the pairs of units which are possibly preferred to each other. In this perspective, SMAA-D may enrich REA with answering questions on how probable are the possible efficiency preference relations and what is the distribution of ranks or efficiencies between the best and the worst ones. These results can be further exploited to indicate the expected rank (efficiency) for a given DMU, the ranks (efficiencies) which are attained most often, and the probability of being judged as efficient (obtaining the highest efficiency).

On the other hand, even though the stochastic indices can be estimated with high accuracy using Monte Carlo simulation, they are not exact. In particular, it may be unlikely to hit the weight vector corresponding to the extreme results. This, in turn, implies that such results would not be reflected in the distribution of ranks or efficiency scores. For the same reason, an estimated pairwise efficiency outranking index equal to one or zero does not, respectively, confirm the necessity or exclude the possibility of one DMU being preferred over another. Still, all these input/output

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