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Evaluating productive performance: A new approach based on the product-mix problem consistent with Data Envelopment Analysis ☆

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ABSTRACT

We propose a new approach to estimate technical coefficients from a set of Decision Making Units (DMUs) under the assumption that their production plans are set by process engineers through Linear Programming (LP) techniques. The idea behind this approach is that most manufacturing and agricultural firms routinely resort to LP-based modeling in their decision making processes in order to plan output production and, therefore, this particularity should be taken into account when estimating their technical efficiency. A usual model of LP for these sectors is the so-called product-mix problem, which we relate to a standard Data Envelopment Analysis (DEA) model in terms of the Directional Distance Function. In this paper, we finally show how to estimate the technical coefficients of a sample of Andalusian farms in Spain and how this information can be seen as a complement to the usual by-products associated with estimating technical efficiency by DEA.

1. Introduction

Efficiency evaluation in production of whatever type of private firm or public organization has been a relevant topic for managers and policy makers, as well as an area of interest from a practical and methodological point of view in both engineering and economics [13,18]. The main aim of such assessment is to analyze the efficiency of a set of observations: generally termed DMUs (Decision Making Units) that use several inputs to produce several outputs, by comparing their performance with respect to the boundary of a production possibility set, and using to that end a sample of other observations operating in a similar technological environment. In the case of producing only one output the interest lies with the notion of production function, which represents the maximum product obtainable from the input combination at the existing state of technical knowledge. The usual methods for measuring technical efficiency of production need explicitly or implicitly to determine the boundary of the underlying technology, which constitutes the reference benchmark. Its estimation allows calculating the corresponding technical inefficiency value for each DMU as the deviation of each activity or production plan to the set of optimal ones, represented by the frontier of the production possibility set or, if it is the case, by the industry production function.

Regarding the determination of the technology in practice, before Farrell's [17] seminal contribution, economists used to specify parametrically the corresponding production functions, e.g., a Cobb-Douglas function [12], relying on Ordinary Least Squares (OLS) regression analysis to estimate an 'average' production function, and assuming that disturbance terms had zero mean. This was a patently unsatisfactory estimation, as it did not follow the traditional (frontier) definition of production functions in microeconomics as the maximal feasible output for each input combination considered. Farrell [17] was the first in showing, for a single output and multiple inputs, how to estimate an isoquant enveloping all the observations. He based his significant contribution on the construction of a production possibility set that satisfied two usual axioms: convexity and monotonicity. In this way, the most conservative estimation of the production possibility set may be obtained through the determination of the minimal set that envelops the observations and, at the same time, meets the two aforementioned axioms [16, p. 255]. Farrell's principle of conservation, now known as 'minimal extrapolation', leads to the estimation of a piece-wise linear isoquant in the input space. For the application of his method to a dataset from the US agricultural sector, Farrell resorted to finding out all the facets of the

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piece-wise linear frontier by applying more or less sophisticated combinatorial methods. The work of Farrell represented an enormous advance in the measurement of efficiency of production by showing how to decompose cost efficiency into technical and allocative counterparts. His contribution constitutes the first implementation of Debreu's coefficient of resource utilization [14] and Shephard's input distance function [35].

Although Farrell also dealt with the possibility of approximating a Cobb–Douglas production function from the previously estimated piece-wise linear isoquant, as a way of summarizing in a few parameters the main features of the underlying technology, Farrell's approach can be categorized in the current area of non-parametric techniques (see Diagrams 4, 5, 7 and 8 in [17]), since it is not necessary to identify *a priori* the specific mathematical formulation of the industry production function to be estimated. This line of research, initiated by Farrell, was later taken up by Charnes et al. [9] and Banker et al. [7], resulting in the development of the Data Envelopment Analysis (DEA) approach, in which the determination of the frontier is only restricted via its axiomatic foundation, mainly convexity and minimal extrapolation. Another paper working in this same line, is that by Afriat [2], showing how to determine a production function with the property P (e.g., non-decreasing concavity) that represents the set of observations to be as nearly efficient as possible. Under the production of only one output, the estimated production functions suggested by Afriat coincide with those associated with what later was known as DEA. A more natural sequel than the DEA literature of the previous work done by econometricians, even before Farrell's contribution, would be Aigner and Chu [1], who show how to apply a technique based on mathematical programming to yield an envelope 'parametric' Cobb–Douglas production function by controlling the sign of the disturbance terms and, therefore, allowing to make the traditional interpretation of the production function in microeconomics.

All the aforementioned methods for estimating the underlying technology from a data sample can be applied to any set of profit and not-for-profit organizations, from a farm to a university, without considering the specific methodologies and techniques that may have been used by managers to determine resource allocation and output mixes in their production processes. Indeed, these methods only depend on observing the inputs and outputs quantities and prices of each sample unit, as these were originally designed to overcome the lack of information (and the cost of collecting it) on what happens within each organization; i.e., they consider production processes as 'black boxes' where the intermediate transformation of inputs into outputs is not specified. However, some attempts to develop new methodologies for opening the black box have been recently published, mainly in the non-parametric field. Examples of these contributions are, on the one hand, those based on network DEA, which considers each unit as composed of distinct processes or stages, each one with its own inputs and outputs and with intermediate flows among the stages (Prieto and Zofio [34], Tone and Tsutsui [37] and Kao [23], to name but a few) and, on the other hand, the recent approach by Cherchye et al. [11], which explicitly includes information about the allocation of the output-specific inputs to their outputs; information that is collected from the activity-based costing (ABC) system of the firms, when it is available.

Ultimately, the way the firm internally plans the output mix has been usually neglected in the mainstream literature on production theory, being an issue that merits further research. This paper is devoted to making progress in this direction by showing the link between the product-mix problem (PMP) and DEA. While the former allows the characterization of the production technology *within* the firm at the individual level, the latter concerns the estimation of the boundary of the reference technology by comparing production plans *between* firms. In doing so, we propose a new approach based on the PMP that will take into account resources and technical coefficients. A recent survey showed that most Fortune 500 companies regularly use linear programming (LP) in their decision making [30]. A typical application of LP is the formulation and resolution of the product-mix problem. Process engineers worldwide have been trained to model such problems through the application of techniques of Operations Research.

A simplified example of a real-life situation of this type, represented by a firm producing leatherwork, taken from Winston [39], is adopted in the following section to illustrate the starting point of the new approach. Given output prices, input and output quantities, and the technical coefficients relating them, the product-mix problem is used to maximize revenue choosing the amount of output quantities to produce, subject to the inputs constraints. The information used for building these constraints are, on the one hand, resource endowments and, on the other, the technical coefficients, which represent the amount of each resource that are consumed in the production of one unit of each output product. In this way, the technology that is utilized for determining the firm's optimal output mix is given by the constraints of the PMP model. We show that it is possible to use the PMP to jointly determine a common technology[1] for a set of observations and estimate the corresponding technical coefficients.

Specifically, at a setting of manufacturing or agricultural production, where LP is a usual tool for determining the optimal output mix, and assuming that we observe a sample of DMUs operating in a similar technological environment, it is our aim to estimate a common set of technical coefficients for all the units associated with the underlying technology. To that end, we invoke Farrell's principle of conservation to determine the technical coefficients that yield the minimal set associated with the 'LP' resource constraints that, at the same time, envelops all the observations. In this way, a polyhedral production possibility set is estimated based on exactly m constraints, denoting by m the number of firm's resources (inputs). The geometrical shape of the estimated technology justifies comparing the new approach to DEA, since by this last technique a polyhedral production possibility set is also estimated based on an *a priori* undetermined number of hyperplanes (greater than the number of inputs with high probability). The distinctive feature of the approach that we introduce is that it contributes to opening the aforementioned 'black box' of efficiency measurement in two manners. First, incorporating into the analysis the usual way in which process engineers internally model the PMP. Second, estimating key information of the firms; in particular how inputs and outputs are linked to each other in the production process.[2]

Therefore, the proposed methodology represents a new approach to assess the production performance of firms that complements the results obtained with the standard DEA approach, as both are closely related. The new approach can be applied by process engineers in many industrial sectors, offering an alternative way to assess productive performance that is grounded on more familiar techniques, but without losing sight of the existing DEA methods. To sum up, we extend the notion of technical and economic efficiency to a wider audience who is

[1] It is commonly adopted in the literature, either implicitly or explicitly, that all firms share the same production possibility set and differ only with respect to their degree of inefficiency (see, e.g., [38,19,31]).

[2] It is also worth mentioning that linear production technologies and the behavior of producers have long been linked in the literature. Owen [33] used cooperative game theory to study a general class of linear production games in which multiple producers using the same technology decide whether to pool their available resources to produce some goods. Many researchers have extended this approach in different directions (see, e.g., Timmer et al. [36] and more recently Lozano [25], which mixes cooperative game theory and DEA).

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