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Multiplicative consistency analysis for interval fuzzy preference relations: A comparative study ☆

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ABSTRACT

To examine the multiplicative consistency of interval fuzzy preference relations (IFPRs), this paper first analyzes the limitations associated with the previous consistency concepts. Accordingly, a new consistency concept is defined that is an extension of the crisp case and overcomes limitations in the previous concepts. Next, a linear programming model to judge the consistency of IFPRs is constructed, and an approach to derive multiplicative consistent IFPRs is introduced. Furthermore, goal-programming models to determine missing values in an incomplete IFPR are constructed that have the highest consistent level with respect to known values. Moreover, a multiplicative consistency and consensus based method for group decision making with IFPRs is developed that can address incomplete and inconsistent cases. Finally, two practical decision-making problems are offered to demonstrate the feasibility and efficiency of the new method, and an analysis of a numerical and theoretical comparison with several related methods is performed.

1. Introduction

Preference relations are efficient tools to address decision-making problems that only require the decision makers to compare one pair of objects at a time. There are two general types of preference relations according to preference values, i.e., numerical preference relations [1–6] and linguistic preference relations [7–12]. Due to the complexity of practical decision-making problems, preference relations with exact variables [1,2,8,9] limit the application of this decision-making method. On the other hand, preference relations with fuzzy numbers [3,4,7] suffer from several issues that have not yet been addressed [13].

Interval preference relations appear to be a good choice to address the above-mentioned issues that permit decision makers to use intervals rather than real numbers to denote their judgments. Similar to traditional preference relations [1,2], there are two types of interval preference relations: interval multiplicative preference relations [14] and interval fuzzy preference relations [15]. Consistency analysis is critical to avoid inconsistent results. With respect to interval multiplicative preference relations, there are primarily four multiplicative consistency concepts [16–19]. Each of them can be considered an extension of the consistency concept provided by Saaty [1], and they each are based on different perspectives. Conversely, there are two types of consistent IFPRs, namely additive consistent IFPRs [20–25] and multiplicative consistent IFPRs [20,21,25–29]. These two types are extensions of consistency concepts introduced by Tanino [6] for reciprocal preference relations. Xu and Chen [20] introduced the concepts of additive consistent IFPRs and multiplicative consistent IFPRs, by which several models to calculate the interval priority weight vector are constructed. Using Xu and Chen's consistency concepts [20], Xu [21] presented two approaches to construct additive and multiplicative consistent IFPRs, respectively. Then, the author established a quadratic programming model to determine the weights of the decision makers and further gave two methods for group decision making with IFPRs. Hua et al. [22] noted the limitation in Xu and Chen's method [20] and built several improved programming models. Chen et al. [23] used the arithmetic mean of interval judgments and Tanino's additive consistency concept [6] to develop an approach for group decision-making with IFPRs. When the group consensus degree of individual IFPRs is smaller than the predefined threshold, an

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interactive method to improve the consensus is introduced. In contrast to above methods to calculate the interval priority weight vector from an identical IFPR, Meng et al. [24] adopted Xu and Chen's additive consistency concept [20] to construct several consistency based models to derive the interval priority weight vector from (incomplete) IFPRs, where the interval priority weights are calculated with respect to different IFPRs. Based on additive and multiplicative consistency analysis, Xia and Xu [30] constructed several programming models to determine the interval priority weight vector from IFPRs. Genç et al. [31] defined the interval fuzzy multiplicative transitivity for IFPRs and used it to develop a method for estimating missing values in incomplete IFPRs, and Wu and Chiclana [32] applied this multiplicative transitivity to develop a multiplicative consistency and trust-consensus based approach for group decision making with IFPRs. It needs to be noted that programming-model based methods [18,20,22,25,27,30,33] fail to address inconsistent IFPRs. Furthermore, as several researchers [22,25,34] noted, it is unsuitable to directly apply the consistency concept to IFPRs for the crisp case.

In contrast to above researches about the consistency for IFPRs, Liu et al. [27] defined a multiplicative consistency concept for IFPRs using a convex combination method. Liu et al.'s multiplicative consistency concept depends on object labels, and different ranking orders can be derived from an identical IFPR only because of different compared orders of objects. Later, Wang and Li [25] proposed an additive consistency concept and a multiplicative consistency concept for IFRPs. The main feature of these two concepts is that they are robust to the compared orders of objects. Using these two consistency concepts, Wang and Li [25] constructed two types of goal-programming models to obtain the interval priority weight vector. One type is based on the additive consistency analysis; the other adopts the multiplicative consistency analysis. It is worth noting that multiplicative consistency based programming model [25] is not equivalent to the defined multiplicative consistency concept. Wang and Chen [28] noted that Xia and Xu's multiplicative consistency concept [35] depends on the compared orders of objects. Then, the authors applied Wang and Li's multiplicative consistency concept [25] to establish a logarithmic least squares optimization model to calculate the interval priority weight vector. Furthermore, Wang and Li [29] developed an approach for group decision making with incomplete IFPRs using Wang and Li's multiplicative consistency concept [25]. However, decision-making approaches in [25,28,29] are insufficient to address incomplete IFPRs because there might be more than one value satisfying the consistency requirement. In particular, different ranking orders can be derived with respect to different estimated values. Furthermore, there might not be a positive interval priority weight vector for completely multiplicative consistent IFPRs. Moreover, Wang and Li's consistency concept [25] cannot discriminate consistency of reciprocal preference relations composed of the lower and upper bounds of intervals in IFPRs, indicating that Wang and Li's consistency concept is not an extension of the crisp case.

IFPRs are an important type of preference relations that have been applied in many fields, such as assessment of military equipment [20], international exchange doctoral selection [25], assessment of the new type of drugs [36], the partner selection of virtual enterprise [37], and the selection of potential suppliers [38]. After reviewing the previous studies on this type of preference relations, we find that there are several limitations, such as consistency, obtaining consistency IFPRs from inconsistent ones, methods to determine missing values, and calculating the interval priority weight vector. In this paper, we continue to discuss IFPRs. First, a new multiplicative consistency concept for IFPRs that is an extension of the consistency concept for reciprocal preference relations [6] is introduced. Furthermore, the paper addresses limitations in the previous multiplicative consistency concepts [25,27,31,35]. Based on the new consistency concept, a model to judge the multiplicative consistency of IFPRs is established, and goal-programming models to estimate missing values are constructed. Then, a multiplicative consistency and consensus based group decision-making method with IFPRs is developed that can address inconsistent and incomplete cases. To accomplish these goals, the paper is organized as follows.

Section 2 initially reviews the concept of IFPRs and then lists two multiplicative consistency concepts. Then, we analyze the limitations of the two concepts. Section 3 introduces a new multiplicative consistency concept. Note that this consistency concept satisfies all of the properties for crisp reciprocal preference relations, and can address the limitations in previous ones. Section 4 describes the construction of a model to judge the consistency of IFPRs and then introduces a method to derive multiplicative consistent IFPRs from inconsistent ones. Meanwhile, multiplicative consistency-based models to determine missing values are built. Section 5 proposes a method of group decision making with IFPRs that is based on multiplicative consistent IFPRs and consensus analysis. Section 6 offers two numerical examples to show the feasibility of the developed method and a comparison analysis is performed. The conclusions and remarks are provided in the last section.

2. Basic concepts

This section reviews several basic concepts, such as intervals, reciprocal preference relations and IFPRs. Meanwhile, several operational laws on intervals are provided, which are used in the following sections. Then, it offers two multiplicative consistency concepts [25,27] and analyzes their limitations from theoretical and numerical aspects.

Let $\Re$ be the set of all real numbers; $\overline{a}=[a^-,a^+]$ is an interval number if $a^- \le a^+$ with $a^-,a^+ \in \Re$, and $\overline{a}$ is a positive interval number if $a^- \le a^+$ with $a^- > 0$. Let $\overline{a}=[a^-,a^+]$ and $\overline{b}=[b^-,b^+]$ be any two positive intervals. From Minkowski operations on intervals, we derive the following:

(*i*) $\overline{a} \oplus \overline{b} = [a^- + b^-, a^+ + b^+]$;

(ii) $\overline{a} \otimes \overline{b} = [a^- b^-, a^+ b^+]$;

(iii) $\overline{a}/\overline{b} = [a^-/b^+, a^+/b^-]$;

(iv) $\lambda\overline{a} = [\lambda a^-, \lambda a^+] \quad \lambda \ge 0$.

To denote decision makers' uncertain judgments on compared objects, interval fuzzy preference relations introduced by Xu [15,39] are good choices.

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