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Common benchmarking and ranking of units with DEA☆

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ABSTRACT

This paper develops a common framework for benchmarking and ranking units with DEA. In many DEA applications, decision making units (DMUs) experience similar circumstances, so benchmarking analyses in those situations should identify common best practices in their management plans. We propose a DEA-based approach for the benchmarking to be used when there is no need (nor wish) to allow for individual circumstances of the DMUs. This approach identifies a common best practice frontier as the facet of the DEA efficient frontier spanned by the technically efficient DMUs in a common reference group. The common reference group is selected as that which provides the closest targets. A model is developed which allows us to deal not only with the setting of targets but also with the measurement of efficiency, because we can define efficiency scores of the DMUs by using the common set of weights (CSW) it provides. Since these weights are common to all the DMUs, the resulting efficiency scores can be used to derive a ranking of units. We discuss the existence of alternative optimal solutions for the CSW and find the range of possible rankings for each DMU which would result from considering all these alternate optima. These ranking ranges allow us to gain insight into the robustness of the rankings.

1. Introduction

Data Envelopment Analysis (DEA) evaluates performance of decision making units (DMUs) involved in production processes. In DEA, the DMUs are assumed to be comparable (see Dyson et al. [20] for a discussion on homogeneity assumptions about the units under assessment), albeit they may have their own unique circumstances. Specifically, DEA may allow for the individual circumstances of the DMUs through DMU-specific input and output weights. It is usually argued that, aside from the factors affecting performance considered in the efficiency analysis, there are often considerable variations in goals, policies, etc., among DMUs, which may justify the different weights for the same factor. The variation in weights in DEA may be thus justified by the different circumstances under which the DMUs operate, and which are not captured by the chosen set of inputs and outputs factors (see Roll et al. [34] for discussions).

However, in many DEA applications the DMUs experience similar circumstances, so treating them as independent entities may not be appropriate. This means that using input and output weights that differ substantially across DMUs may not be warranted. In those situations, both inputs and outputs should be aggregated by using weights that are common to all the DMUs when calculating the classical efficiency ratios. As stated in Roll et al. [34], common set of weights (CSW) "*is the usual approach in all engineering, and most economic, efficiency analyses. In these cases it is assumed that all important factors affecting performance are included in the measurement system, and there is no need (nor wish) to allow for additional, individual, circumstances.*" In addition to the appeal of a fair and impartial evaluation, in the sense that each variable is attached the same weight in the assessments of all the DMUs, it should be noted that, unlike DEA, CSW allows us to rank the DMUs. The fact that DEA uses different profiles of weights in the assessments of the different DMUs makes impossible to derive an ordering of the units based on the resulting efficiency scores (see Cooper and Tone [15], Sinuany-Stern and Friedman [40], Kao and Hung [23] and Ramón et al. [30] for discussions). Moreover, poor discrimination is often found in the assessment of performance with DEA models, since many of the DMUs are classified as efficient or are rated near the maximum efficiency score. This can also be alleviated with a CSW. See Adler et al. [1], which provides a survey of ranking methods in the context of DEA. See also Angulo-Meza and Estellita Lins [4] and Podinovski and Thanassoulis [28], which review the problem of improving discrimination in DEA.

The choice of a CSW may often raise serious difficulties (see Doyle and Green [19] for a discussion). In particular, DEA has been used to find a CSW, specifically as the coefficients of a supporting hyperplane of the DEA technology at some efficient DMUs. In

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many cases, the choice of such hyperplane is made by minimizing the differences between the DEA efficiency scores and those that would result from the associated CSW. See Despotis [17], Kao and Hung [23], Liu and Peng [25,26] [1] and Cook and Zhu [14] (in the latter paper the objective is relaxed to groups of DMUs which operate in similar circumstances). Ganley and Cubbin [22] and Troutt [46] also find CSWs by somehow maximizing the resulting efficiency scores: maximizing the sum of efficiency ratios of all the DMUs, in the former case, or maximizing the minimum efficiency ratio, in the latter[2]. These DEA-based approaches utilized to derive a CSW, in spite of using a reference production technology, are only concerned with the measurement of efficiency and the subsequent ranking of DMUs. However, in this context, it may be argued that the DMUs at which the hyperplane (associated with the CSW) supports the technology can also be used for purposes of benchmarking and target setting, in a similar manner as they are somehow used as referents when efficiency scores based on that CSW are calculated. Specifically, the facet of the DEA efficient frontier that those DMUs span could be seen as a common best practice frontier for the benchmarking.

In contrast to the procedures used to the choice of a CSW mentioned above, this paper is primarily focused on the development of a common framework for the benchmarking. Nevertheless, the DEA-based approach we propose also provides a CSW as an additional product, which can be used for the measurement of efficiency and ranking. In DEA, the efficient DMUs form a piecewise linear frontier that can be seen as a best practice frontier in the circumstance of benchmarking (see [13]). See Thanassoulis et al. [43] for a discussion on the issue of benchmarking in DEA; and also Cook et al. [12], Adler et al. [2] and Dai and Kuosmanen [16] for some references on DEA and benchmarking which include applications. Here, it is assumed that we deal with a situation in which there is no need (nor wish) to allow for individual circumstances of the DMUs. Therefore, as discussed above, the DMUs should be evaluated with input and output weights that are common to all of them. Likewise, it is also reasonable to assume that they should have common benchmarks and establish common best practices. From a methodological point of view, if a DEA-based approach is used for the benchmarking, assuming common weights means that only a facet of the DEA efficient frontier should be considered as the best practice frontier. The common best practice frontier will be therefore the facet of the DEA efficient frontier spanned by a set of technically efficient DMUs, which can be seen as a common reference group. Targets will result from projections of the DMUs on to this common best practice frontier. We select the common reference group as that which provides the closest targets. Minimizing the gap between actual inputs/outputs and targets ensures the identification of best practices that are globally the most similar to the actual performances of the DMUs being evaluated. Thus, they may show the DMUs the easiest way for improvement.

The fact that the DMUs are all projected on to the same facet of the efficient frontier means that the setting of targets could involve the deterioration of some observed input/output level (provided that some of the others are improved). That is, efficient targets might suggest that improvements can be accomplished by reallocations between inputs and/or outputs. In other words, in the common benchmarking approach dominance does not prevail. As a consequence, although the approach proposed is developed in a DEA context, we depart here from the notion of technical efficiency used in the standard DEA. This situation is similar to that of the DEA models with weight restrictions. Thanassoulis et al. [42] state that in those cases we actually move from the technical efficiency to a kind of overall efficiency. Note also that the scores provided by the CSWs obtained by using the other existing DEA-based methods mentioned above do not measure technical efficiency either, for the same reasons.

Technically, we follow a primal-dual approach based on a model that includes constraints of both the envelopment and the multiplier formulations of the DEA models. Thus, its optimal solutions allow us to deal not only with benchmarking and target setting but also with the measurement of efficiency. In particular, the optimal weights, which are the coefficients of the supporting hyperplane of the technology that contains the common best practice frontier, can be used to define efficiency scores for all the DMUs. As said before, as these weights are common to all the DMUs, the resulting efficiency scores allow us to derive a full ranking of units. This approach is in line with that used in Aparicio et al. [7], which is also concerned with minimizing the distance to the Pareto-efficient frontier of the production possibility set (PPS)[3]. However, that paper provides a self-evaluation of units (regarding technical efficiency), where each DMU can choose its own input and output weights, as opposed to the evaluation of the DMUs made here within a common benchmarking framework. As a result, Aparicio et al. [7] does not address the problem of ranking DMUs (in fact, it does not deal with the measurement of efficiency).

Finally, we note that, if the identified common reference group spans a facet of the DEA efficient frontier which is not of full dimension, we will have alternative optimal solutions for the CSW. To deal with this issue, we propose an approach that considers all these optimal solutions for the CSWs, thus avoiding the need to introduce an additional criterion to the choice of weights among alternate optima. Specifically, a couple of models that allow us to yield for each DMU a range for its possible rankings are developed. These ranking ranges can help us gain insight into the robustness of the rankings.

The paper unfolds as follows: in Section 2, we develop a model that allows us to set the closest targets on a common facet of the DEA efficient frontier. The optimal solutions for the weights of this model yield CSWs that can be used to define efficiency scores and rank the DMUs. These issues are addressed in Section 3, where we also discuss the existence of alternate optima for the CSW and provide ranges of possible rankings for each unit. Section 4 includes an empirical illustration. Section 5 concludes.

2. Closest targets on a common facet of the Pareto efficient frontier

Throughout the paper, we consider that we have n DMUs which use m inputs to produce s outputs. These are denoted by (X_j, Y_j), $j=1,\dots,n$. It is assumed that $X_j=(x_{1j},\dots,x_{mj})'\geq 0$, $X_j\neq 0$, $j=1,\dots,n$, and $Y_j=\left(y_{1j},\dots,y_{sj}\right)'\geq 0$, $Y_j\neq 0$, $j=1,\dots,n$. We also assume a DEA constant returns to scale technology [9] for the measurement of relative efficiency and benchmarking. Thus, the production possibility set (PPS), $T=\{(X,Y)/\ X \text{ can produce } Y\}$, can be characterized as follows $T=\left\{(X,Y)/\ X\geq\sum_{j=1}^{n}\lambda_jX_j,\ Y\leq\sum_{j=1}^{n}\lambda_jY_j,\ \lambda_j\geq 0\right\}$.

The following model simultaneously provides for every DMU the closest targets on a (common) facet of the Pareto efficient frontier of T by minimizing globally a weighted L_1-distance to their

[1] A model similar to the ones proposed in these two papers can be found in Liu et al. [24]. However, that model provides CSWs that are not necessarily DEA weights and may lead to efficiency scores larger than 1.

[2] See also Roll and Golany [35] and Ramón et al. [31] for other papers dealing with DEA and CSWs.

[3] See also Portela et al. [29], Tone [45], Fukuyama et al. [21], Aparicio and Pastor [6] and Ruiz et al. [37] for other papers which deal with closest targets in DEA.