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Optimal coordination of resource allocation, due date assignment and scheduling decisions ☆

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ABSTRACT

We study a single-machine scheduling problem in a flexible framework, where both job processing times and due dates are decision variables controllable by the scheduler. Our objective is to provide a practical tool for managers to optimally (or approximately) coordinate higher level decisions (such as delivery date quotation) with lower level (operational) decisions (such as scheduling and resource allocation). We analyze the problem for two due date assignment methods and a convex resource consumption function. For each due date assignment method, we provide a bicriteria analysis where the first criterion is to minimize the total weighted number of tardy jobs plus due date assignment cost, and the second criterion is to minimize total weighted resource consumption. These bicriteria problems are known to be $\mathcal{NP}$-hard. In this paper, for each due date assignment method, we develop pseudo-polynomial algorithm and fully polynomial time approximation scheme (FPTAS) to minimize the total weighted number of tardy jobs plus due date assignment costs subject to an upper bound on the total weighted resource consumption.

1. Introduction

In most cases firms evaluate the efficiency of manufacturing in terms of operational costs, while marketing is evaluated in terms of revenue (see, e.g., Balasubramanian and Bhardwaj [2], Karmarkar and Lele [25], and Pekgun et al. [38]). Accordingly, higher level decisions such as pricing and delivery date quotation are usually made by the marketing department while operational decisions such as scheduling and resource allocation are done separately in the lower (operational) level of the decision making hierarchy. However, such a decentralized decision making process can lead to suboptimal solutions and may in turn affect firm profitability. In fact, Otley [35] showed how dividing a firm into independent units would lead to misaligned incentives and suboptimal system performance. Likewise, Malhotra and Sharma [32] emphasized the strong need to align manufacturing and marketing decisions with the firm's goals and objectives, and Hausman et al. [14] showed (empirically) that business performance is enhanced when manufacturing and marketing work together for a common goal.

In traditional scheduling models, due (delivery) dates are assumed to be determined exogenously on a higher hierarchical level than the actual scheduling decisions (see, e.g., Sen and Gupta [41], Koulamas [26] and Baker and Scudder [3]). However, the need to better coordinate upper- and lower-level decisions led to an increasing number of works in which due date assignment, resource allocation and scheduling decisions are made simultaneously (see, e.g., Panwalkar and Rajagopalan [37], Alidaee and Ahmadian [1], Cheng et al. [7], Biskup and Jahnke [5], Ng et al. [34], Shabtay and Steiner [44,45,47], Shabtay et al. [48], Leyvand et al. [30,31] and Ji et al. [18,20,21]). In this paper we similarly attempt to improve firm performance by providing a unified framework to optimally coordinate higher level delivery date quotation decisions with lower level operational decisions such as resource allocation and scheduling. We use two of the most common methods to assign due dates for jobs (see, e.g., Seidmann et al. [40], Panwalkar et al. [36], Chen [6], Shabtay and Steiner [42,46], Mosheiov and Yovel [33], Ji et al. [19,22], and Yin et al. [52]):

- The common due-date assignment method (usually referred to as *CON*), for which all jobs are assigned the same due date, that is $d_j = d$ for $j = 1, \ldots, n$, where d_j is the due date of job j and $d \geq 0$ is a decision variable.
- The slack due-date assignment method (usually referred to as *SLK*), for which all jobs are given a flow allowance that reflects equal waiting time (i.e., equal slacks), that is, $d_j = p_j + slk$ for

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$j = 1, \ldots, n$, where p_j is the processing time of job j and $slk \geq 0$ is a decision variable.

For more details on scheduling problems involving due date assignment decisions we refer the reader to the surveys by Gordon et al. [10–12] and Kaminsky and Hochbaum [24].

Like due dates, job processing times in scheduling were traditionally considered to be fixed parameters. In various real-life systems, however, processing times may be controllable by allocating resources, such as additional money, overtime, energy, fuel, catalysts, subcontracting, or additional manpower to the jobs. Vickson [51] was the first to study a shop scheduling problem with controllable processing times. He pointed out that "least cost scheduling through job processing time control has been studied thoroughly in the project management context. In view of the importance of, and familiarity with job processing time choice in project planning models, it is perhaps surprising that similar concepts have received little attention in the sequencing portion of the scheduling literature." Following the impetus of Vickson's paper, sequencing problems with controllable processing times have been extensively studied by various researchers (e.g., Alidaee and Ahmadian [1], Janiak and Kovalyov [16], Cheng et al. [7], Liman et al. [29], Biskup and Cheng [4], Ng et al. [34], Shabtay and Steiner [45] and Leyvand et al. [30,31]). The most recent surveys of scheduling models with controllable processing times are Janiak et al. [17] and Shabtay and Steiner [43].

Our scheduling problem considers a centralized decision making process with the objective of coordinating due date quotation, scheduling and resource allocation decisions. It can be formally presented as follows: n independent, non-preemptive jobs of set $J = \{1, 2, \ldots, n\}$ are available for processing at time zero and are to be processed by a single machine. The processing time of job j, p_j, is a continuous convex function of the amount of resource that is allocated to the processing of the job given by the following resource consumption function:

$$p_j(u_j) = \left(\frac{\theta_j}{u_j}\right)^k, \tag{1}$$

where $u_j \geq 0$ is a decision variable that represents the amount of resource allocated to job j, θ_j is a positive parameter that represents the workload for processing job j and k is a positive constant. This model reflects the law of diminishing marginal returns and has been used extensively in continuous resource allocation theory. The due dates are assignable either according to the CON or the SLK method. A schedule S is determined by a job sequence, a resource allocation vector $\mathbf{u}^* = (u_1, u_2, \ldots, u_n)$, and either a common due date d (CON method) or a common slk value (SLK method). The quality of a schedule S is measured by two criteria: The first one is the scheduling criterion, which includes (weighted) penalties for tardy jobs and the cost of due date assignment, defined by

$$Z(S) = \sum_{j=1}^{n} \alpha_j U_j + \sum_{j=1}^{n} \beta_j d_j, \tag{2}$$

and the second criterion is the total resource consumption cost (or amount) given by

$$V(S) = \sum_{j=1}^{n} v_j u_j, \tag{3}$$

where for schedule S, $C_j(S)$ is the completion time of job j and U_j is the tardiness indicator variable for job j, i.e., $U_j = 1$ if $C_j(S) > d_j$ and $U_j = 0$ if $C_j(S) \leq d_j$. In addition, α_j is the tardiness cost, β_j is the cost of one unit of delivery time quotation, and v_j is the cost of one unit of resource allocated for job j. To simplify the notation, we omit the argument S where it is not necessary. For example, we let V denote the total resource consumption cost in schedule S instead of $V(S)$. Note that Eq. (2) can be converted to

$$Z(S) = \sum_{j=1}^{n} \alpha_j U_j + d \sum_{j=1}^{n} \beta_j \tag{4}$$

and to

$$Z(S) = \sum_{j=1}^{n} \alpha_j U_j + slk \sum_{j=1}^{n} \beta_j + \sum_{j=1}^{n} \beta_j p_j(u_j) \tag{5}$$

for the CON and SLK methods, respectively [44].

Since our problem has two criteria, four different optimization problems can arise for each due date assignment method:

- The first, which we denote by P1, is to minimize $F_l(Z, V) = \sum_{j=1}^{n} \alpha_j U_j + \sum_{j=1}^{n} \beta_j d_j + \sum_{j=1}^{n} v_j u_j$. Using the scheduling notation introduced in [50], this problem can also be referred to as $1|X, conv|F_l(Z, V)$ for $X \in \{CON, SLK\}$;
- The second, denoted by P2, is to minimize $\sum_{j=1}^{n} \alpha_j U_j + \sum_{j=1}^{n} \beta_j d_j$ subject to $\sum_{j=1}^{n} v_j u_j \leq V$; where V is a limitation on the total resource consumption cost. Following the notation in [50], we refer to this problem as $1|X, conv|\epsilon(Z/V)$ for $X \in \{CON, SLK\}$;
- The third, which we denote by P3, is to minimize $\sum_{j=1}^{n} v_j u_j$ subject to $\sum_{j=1}^{n} \alpha_j U_j + \sum_{j=1}^{n} \beta_j d_j \leq K$, where K is a given upper bound on the scheduling cost. We refer to this problem by $1|X, conv|\epsilon(V/Z)$ for $X \in \{CON, SLK\}$ (based on [50]);
- The fourth, P4 (and referred to by $1|X, conv|\#(V, Z)$ for $X \in \{CON, SLK\}$), is to identify each Pareto-optimal point and its *Pareto-optimal schedule*, where a schedule S with $V = V(S)$ and $Z = Z(S)$ is called *Pareto-optimal* (or *efficient*) if there does not exist another schedule S' such that $V(S') \leq V(S)$ and $Z(S') \leq Z(S)$ with at least one of these inequalities being strict. The corresponding Pareto-optimal point is (V, Z).

It should be noted that solving P4 also solves P1–P3 as a by-product. Note also that the decision versions (*DVP*) of problems P2 and P3 are identical, as they both ask if there exists a schedule with $\sum_{j=1}^{n} \alpha_j U_j + \sum_{j=1}^{n} \beta_j d_j \leq K$ and $\sum_{j=1}^{n} v_j u_j \leq V$.

A linear time optimization algorithm was previously presented in Shabtay and Steiner [44] for the P1-type scheduling problems $1|CON, conv|F_l(Z, V)$ and $1|SLK, conv|F_l(Z, V)$. By reducing the *Partition* problem to *DVP* for both the *CON* and the *SLK* due date assignment methods, Shabtay and Steiner [47] later proved that the $1|X, conv|\epsilon(Z/V)$ and $1|X, conv|\epsilon(V/Z)$ problems are $\mathcal{NP}$-hard for both X=*CON* and X=*SLK*. However, the question of whether these problems are strong or ordinary $\mathcal{NP}$-hard remained open. In this paper, we answer these issues for the P2-type problems, $1|CON, conv|\epsilon(Z/V)$ and $1|SLK, conv|\epsilon(Z/V)$, by providing pseudo-polynomial algorithms and fully polynomial time approximation schemes (*FPTAS*).

The rest of the paper is organized as follows. In Sections 2 and 3, we show that the $1|CON, conv|\epsilon(Z/V)$ and the $1|SLK, conv|\epsilon(Z/V)$ problems reduce to a constrained 2-partition and 3-partition problem, respectively. These partition problems have very complex non-linear objective functions, which makes it hard to design dynamic programming recursions for them. Nevertheless we were able to construct recursions to solve these two problems in pseudo-polynomial time by using the state-space generation method. Moreover, we succeeded to convert the pseudo-polynomial time algorithms into FPTASs, by innovatively subdividing the solution spaces into a polynomial number of auxiliary constrained problems. We note that the construction of FPTASs for non-linear combinatorial problems is not an easy task, and our approach deals with problems similar to some problems presented in Halman et al. [13]. However, in contrast to the scheme in [13], our FPTAS's fall into the category of fast schemes that require

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