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Stock rationing under a profit satisficing objective ☆

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ABSTRACT

This paper addresses a rationing problem with a profit satisficing objective in a company operating many retail stores through a centralized procurement. General rationing problems arise when the available stock or capacity cannot guarantee the possibility to satisfy the demand in full, and different decisions about the allocation of the available resources may lead to different profit results. Therefore, the appropriate allocation of the stock or capacity can have a substantial impact on the company's profit. Unlike other works in the rationing area, this paper considers a profit satisficing objective, which entails maximizing the probability of achieving a pre-specified profit target. This type of objective is sometimes preferable to maximizing the expected profit. The problem is modeled in an analytical form, for which closed-form solutions are extremely hard to compute. Thus, the conditions for achieving the satisficing objective are discussed, and two heuristic procedures are compared: one exploiting the structure of the problem and resulting in a greedy, marginal unit allocation; the other, based on the Nelder–Mead derivative-free method.

1. Introduction

Managing and allocating scarce resources is an emerging problem in many supply chain contexts. In particular, allocating common resources, such as the product stock or the production capacity, to competing activities or actors with uncertain outcomes and needs is a recurring challenge in many business settings [1]. In this respect, a rationing problem arises whenever the available stock or capacity does not guarantee the possibility to satisfy the demand in full, and different decisions about the allocation of the available resources may lead to different profit results. In this case, the decision maker in the company has to discern, according to the relative importance of the served customers [2], which orders should be filled (either partially or completely) and which orders should be rejected, to achieve a pre-specified objective, such as profit maximization. Thus, in shortage situations, customers are put "on allocation", and the available resources are distributed according to the appropriate rules [3].

In this paper, we consider a rationing problem arising in the context of a company that manages the procurement of a scarce resource (i.e., a product) through a centralized purchasing department and then distributes the resource to many stores that, in turn, sell it to the final market. This may be the case for large distribution firms (i.e., Wal-Mart, 7-Eleven, and Leroy Merlin) that procure the products at a centralized level and then allocate them to their retail stores. In this context, the necessity for solving a rationing problem may be linked to the characteristics of the resource (i.e., a scarce resource), to temporary situations that limit the availability of the resource (i.e., a supplier disruption), or to other constraints that affect the possibility of procuring larger quantities.

Each retailer contributes in varying degrees to the company's bottom line. Incorrect allocation may lead to situations in which some retailers are out of stock, while others have excessive, unsold stock. Thus, the company must carefully decide how to allocate the available stock among the retailers to pursue a given objective.

The main contribution of this paper is to provide analysis and insight into the rationing problem when companies pursue a profit satisficing objective, namely, the objective of maximizing the probability of achieving a pre-specified profit target.

Considering the analytical complexity of the problem, no closed-form solution may be available. Therefore, we introduce and discuss two solution approaches based on marginal analysis, numerical optimization and a derivative-free search method, namely, the Nelder–Mead algorithm. To address these aspects, the paper is organized as follows: In Section 2, we briefly discuss the background of the problem addressed, while in Section 3 the problem is formulated. In Section 4, we discuss the *satisficing conditions*, that is, the conditions that should be met to attain a pre-specified objective, while in Section 5 we illustrate the

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heuristic procedures aiming to solve the maximization problem. We then report the results of numerical testing in Section 6, discussing the benefits and drawbacks of the two procedures. Finally, in Section 7, we report the conclusions, discuss the limitations, and propose future directions for improvement.

2. Background

In a context in which a centrally procured scarce resource must be distributed to different retail stores to reach the final market, the allocation decision is particularly relevant in situations in which the central decision maker has only one opportunity to allocate the stock (i.e., single-period problems) because of the brevity of the selling season or the shortness of the product shelf life (i.e., style goods, fashion apparel, Christmas toys, and dairy or perishable products). In such contexts, there may be no future opportunities to recover from incorrect allocations.

In allocating a limited stock, a company may pursue various objectives. For example, Balakrishnan et al. [4] argued that the objective in capacity rationing problems in make-to-order manufacturing firms is rather similar to the objective in perishable asset revenue management (PARM) problems, which is typical of the service operations management field. In any situation of fixed capacity and a perishable service or product, firms want to avoid spoilage of the service or product, pursuing the objective of realizing the most revenue possible while facing an uncertain demand [5]. Papier and Thonemann [6] considered the case of a rental company offering two service levels (classic and premium) entailing different prices and types of delivery guarantee. In this setting, the company has to decide under which conditions it should ration its limited fleet capacity to classic customers with the objective of increasing the service level of premium customers while meeting the guarantee. Pinto [7] discussed an expected profit maximization case, modeled as a newsvendor-like problem, presenting a multi-step algorithm for stock allocation with service level constraints, while Klein and Kolb [8] considered a firm that wants to optimally allocate fixed and limited capacity to heterogeneous customer segments with the aim of maximizing customer equity, defined as the total value of all current and future customers. Through a Markov decision process formulation, the authors studied the trade-off between short-term attainable revenues and long-term customer relationships. Further examples of objectives are given in the context of centralized decision-making by Fang [9], who illustrated a new approach for performing resource allocation based on efficiency analysis with the aim of optimizing the operations of all of the units, simultaneously reducing the total inputs on a global basis, and Karsu and Morton [10], who discussed the balanced distribution of a common resource, providing a bi-criteria framework to think about trading balance off against efficiency. Finally, Turgay et al. [11] formulated a robust stochastic dynamic program to investigate the maximization of the expected total profit in a system serving different customer classes assuming demand and production rates characterized by uncertainty.

The maximization of the expected profit is probably one of the most common and perhaps intuitive optimization objectives adopted, as it involves cost minimization [12] that represents a necessary but not sufficient condition for profit maximization. In general, the expected profit maximization objective is used because it is based on risk neutrality [13].

Unlike the previous cases, in this paper, we address the rationing problem using the objective of *profit satisficing*, namely, the objective of maximizing the probability of achieving a prespecified profit target [13,14].

The reasons motivating this assumption stem from the realization that, in many managerial situations, a budgeted profit is established, and the disutility resulting from not achieving this targeted profit level is much larger than the rewards for overachieving. A manager may then be interested primarily in maximizing the probability of meeting the budget, regardless of whether the target level is exceeded or barely attained [14]. As argued in [15], budget attainment is sometimes a more accurate characterization of the decision-making process.

Profit satisficing is not a new topic: earlier contributions date back to the 1950s, with the often-cited works by Lanzillotti [16] and Simon [17]. Although different contributions can be found addressing the satisficing objective in different contexts such as inventory placement [18], newsboy-like settings [19,20,13], and supply chain contracts [21–23], to the best of the authors' knowledge, this objective has never been considered in stock rationing. Therefore, it represents the major contribution of this paper.

3. Problem formulation

In this section, we outline the problem formulation providing the necessary assumptions. To this end, consider a company that distributes a single product to the final market through a commercial channel composed of $n \geq 2$ proprietary retailers. The company implements a centralized procurement strategy, and has a stock of A units to be distributed completely to the retailers before the selling season starts. We consider a single-period setting, in which the allocation decision cannot be revised during the selling season. We also exclude the possibility of partial allocation followed by a second allocation after the demand is revealed. Due to these assumptions, it is rational to distribute the whole quantity A among the retailers because each unit kept at the central level will not contribute to attaining the company's satisficing objective.

The retailers sell their allotted quantity to the market until depletion, and the unmet demand is lost. Each retailer sells the product to the final market at a fixed, exogenous price p known in advance (at least, just before the selling period). We assume a unique selling price p for all retailers as common for the context and the type of company under discussion.

The cost of each unit for the company is c. At the end of the selling season, the unsold quantities are scrapped. For the sake of simplicity of exposition and without loss of generality, we assume no salvage value for the leftovers and no stock-out (goodwill) cost. We also assume that the transportation costs to the retailers are non-differential or negligible. For products with a short life cycle, the stock-holding costs are assumed to be negligible.

Each retailer i faces a stochastic demand D_i with known density f_i and cumulative distribution F_i defined in $[0, \alpha_i)$ (or $[0, \alpha_i]$) to exclude negative demand.

Further, we assume that the demands are independent because retailers enjoy local monopolies due to their geographical dispersion and are not in direct competition. These assumptions are commonly supported in the considered problem setting because the company has control over the location of its retailers; therefore, the company would not locate the retailers too close to each other. In this study, we do not explicitly consider the presence of competitor retailers from other companies. Alternatively, we can consider that the demand distributions f_i already account for the presence of competitors in the region.

We define the allocation of the stock A among n retailers as a real-valued vector $\boldsymbol{Q} = (Q_1, \dots, Q_n)$, where $Q_i \geq 0$ is the quantity allotted to the ith retailer. We assume that a simple rounding of values in $\boldsymbol{Q}$ does not introduce significant deviations from optimality. This assumption especially holds when the components of $\boldsymbol{Q}$ are not too small (i.e., few units) and the price is not