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Inventory and transshipment decisions in the rationing game under capacity uncertainty ☆

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ABSTRACT

In this paper, we consider the inventory decisions of two retailers who are supplied by a single supplier with uncertain capacity. When capacity is allocated in proportion to the retailers' orders, the retailers compete for the capacity by inflating their orders (i.e., the rationing game). In addition, we allow the retailers to implement transshipment between them such that they cooperate by transshipping the surplus stock of one to another who is out of stock. Our analysis of Nash equilibrium orders shows that, while order inflation in the equilibrium orders persists in the rationing game with transshipment, it may not occur if the amount of capacity shortage is small and the transshipment prices are low. Thus, carefully chosen transshipment prices may alleviate order inflation behavior. We also characterize centralized orders that maximize the total profit of the retailers and compare them to equilibrium orders. In particular, we investigate coordinating transshipment prices that induce the retailers to choose centralized orders. Our numerical analysis shows that, even for two identical retailers, coordinating transshipment prices exist in a more limited range of parameter values in the rationing game than they do outside the rationing game due to capacity uncertainty and limitation.

1. Introduction

It is not rare in a supply chain that a supplier has insufficient capacity to meet the total demand of retailers [8]. This is more likely to occur when the market is in the growth phase [16] or when it is hard to change the capacity once a capacity decision has been made [2]. Not only may the capacity be limited to satisfy total demand, a capacity shortage situation may occur unexpectedly. In fact, uncertainty during the production process leads to the situation where the shortage of capacity happens randomly. For example, Ciarallo et al. [7] pointed out that increasing the complexity of manufacturing processes and machinery in production generates uncertainty in the supplier's capacity. Thus, the supplier's capacity or yield uncertainty in capacity allocation has been a popular topic in the literature, for example, Hu et al. [13,14].

To deal with the situation where a supplier fails to satisfy the total orders from the retailers, suppliers usually pre-announce an allocation rule to the retailers that determines how to allocate her capacity in a capacity shortage situation. One of the widely used allocation rules in practice is the proportional allocation rule [6], which distributes capacity in proportion to retailers' orders. Other allocation rules have been considered, including a turn-and-earn allocation in the automobile industry [18,8], and linear and uniform allocations [2]. However, the proportional allocation rule is the most intuitive and simple allocation rule [2,3], as it can be easily implemented and used in practice [19]. Under the proportional allocation rule, it is well known that the retailers are tempted to inflate their orders to obtain their desired amounts from the supplier [2]. If each retailer anticipates these temptations, the retailers will compete for the supplier's capacity by inflating their orders [16,2,3]. This situation is referred to as the rationing game [16].

While the retailers in the rationing game care only about their individual performance, they may also act in a mutually beneficial manner. Specifically, one retailer with surplus stock can transfer his stock to an out-of-stock retailer at a transshipment price and obtain an excess profit. If the stock transshipped from this retailer costs less and is obtained more quickly than the stock replenished directly from the supplier, it will also be desirable for the out-of-stock retailer to accept the transshipment offer. The transfer of stock between supply chain members at the same echelon level, referred to as lateral transshipment or transshipment, is a way to better match supply with demand and reduce inventory risk accordingly [11,34].

In many industries, product life and sales cycles become shorter due to the rapid development of technology, severe market competition, and customers' diverse needs [33]. It is then reasonable to observe transshipment between retailers while they are in the rationing game and compete for the capacity. For example,

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in the fast-fashion industry, Zara's retail store managers tend to order top-selling articles exceeding their true needs, especially when speculating that the warehouse may not keep enough inventory to satisfy all stores, and then they use the transshipment practice with other stores when stocked either more or less than expected [5]. If, however, retailers anticipate a persistent supply shortage, the retailer with surplus stock may have an incentive to reject a transshipment request from an out-of-stock retailer and prepare for the next rationing game by reserving its surplus stock. We thus restrict our attention to a single-period setting where transshipment is always beneficial to both retailers [15].

While the rationing game and transshipment have been popularly observed together in practice, to the best of our knowledge, research on the strategic behavior of retailers in the rationing game with transshipment has not appeared in the literature yet. Thus, in this paper, we aim to fill this gap by analyzing the rationing game where independent retailers compete with each other for the supplier's uncertain and limited capacity while they also engage in mutually beneficial transshipment between them. More precisely, in our model, two retailers may manipulate their orders under competition for the supplier's capacity, and after satisfying their own demands with their own stock, may implement transshipment as per prearranged mutual agreement. Our model is based on the rationing game model of Lee et al. [16] and the transshipment models of Rudi et al. [26] and Hu et al. [13].

We assume that the supplier's uncertain and limited capacity follows the Bernoulli distribution with parameter P, (“shortage probability”), and $\underline{k}$, (“limited capacity”). That is, with the probability of P, the supplier fails to satisfy the retailers' total orders by producing $\underline{k}$, which is assumed to be less than the sum of the equilibrium order quantities under infinite capacity. On the other hand, with the probability of $(1-P)$, the supplier produces a sufficient amount to satisfy the retailers' total orders. Our assumption reflects the reality that the supplier usually builds enough capacity to satisfy the desired orders fully from the retailers, while sometimes, due to yield uncertainty, it is not always guaranteed. Examples that cause such interruption include machine breakdown, material shortage, unreliable supply, natural disasters, and labor strikes. See, for example, Anupindi and Akella [1], Dada et al. [9], Haisheng et al. [12], Silbermayr and Minner [29], and Schmitt et al. [27]. Under the supplier's capacity uncertainty, we analyze how retailers adjust their orders in response to change in the shortage probability (P) and in a limited capacity ($\underline{k}$) under the shortage situation.

In our model, the information regarding the supplier's capacity is known not only to the supplier, but also to the retailers. Thus, the retailers know in advance explicitly how likely the supplier will fail to satisfy their total orders and how large the capacity will be in the capacity shortage situation. We then analyze the order strategy of retailers and examine whether a unique Nash equilibrium exists in the rationing game without and with transshipment. Under the assumption that capacity follows the Bernoulli distribution, we prove that, in the rationing game without transshipment, a unique equilibrium exists. In the rationing game with transshipment, a unique equilibrium exists within a certain range of values for the two parameters regarding uncertain capacity: shortage probability and limited capacity. Then, we investigate how the equilibrium orders change in accordance with the probability of capacity shortage (i.e., shortage probability) and the size of the realized capacity in the shortage situation (i.e., limited capacity).

While it is well known that retailers manipulate their orders by inflating them to obtain more allocation from the supplier in the rationing game, transshipment between retailers may reduce such order inflation behavior. We found that, in the rationing game, the retailers with transshipment can strategically order even less than their desired amounts under infinite capacity, especially when the amount of capacity shortage is small and the transshipment prices are low (low enough that they are close to the sum of the salvage value of inventory and transshipment cost). This result suggests that carefully chosen transshipment prices can alleviate order inflation behavior in the rationing game. Also, this implies that transshipment can be an effective way for the retailers to hedge the risk from a supplier's uncertain capacity in the rationing game.

Another research question is motivated by Hu et al. [13], who address existence of predetermined transshipment prices that induce retailers to make centralized order decisions (i.e., coordinating transshipment prices). Based on Hu et al. [13], we study how two retailers' competition in the rationing game affects the existence of coordinating transshipment prices. A unique pair of transshipment prices can be chosen so that the retailers maximize their joint profits [26]. However, such a pair of coordinating prices is less likely to exist as the retailers become more non-identical in terms of cost and revenue parameters [13]. We find that coordinating transshipment prices are even more unlikely to exist when retailers play the rationing game. This finding is also shown by the fact the sufficient condition for the existence of coordinating transshipment prices is more restrictive in the rationing game than it is in the case of infinite capacity due to capacity uncertainty and limitation.

The rest of the paper is organized as follows. In Section 2, we review the related literature. We describe out model in Section 3. In Section 4, we examine the existence and uniqueness of a pure-strategy Nash equilibrium. In Section 5, we analyze retailers' strategic order decisions by focusing on the properties of the equilibrium orders. We characterize the centralized order decisions in Section 6, and we compare the centralized orders with the equilibrium ones in Section 7. In Section 8, we examine the effect of the rationing game on the existence and magnitude of coordinating transshipment prices. We conclude the paper in Section 10 and suggest some future research directions.

2. Literature review

This paper is related to the literature on the rationing game, capacity allocation, and transshipment. The first two research streams are closely related to each other since they consider the setting where retailers make strategic order decisions to obtain better allocation from their supplier. However, while the literature on the rationing game focuses on retailers' ordering behavior, the latter focuses on which allocation rule works best for supply chains. Thus, our work is more closely related to the literature on the rationing game than it is to the literature on capacity allocation. The rationing game model of Lee et al. [16], the base model for our work, considers multiple retailers supplied by a single supplier who distributes her uncertain capacity by the proportional allocation rule. Lee et al. [16] show that under this allocation rule, retailers inflate their orders under the capacity uncertainty of a common supplier. While Lee et al. [16] only show the inflating order behaviors of retailers, we extend this work in this paper by characterizing explicitly the equilibrium orders in the case of two retailers and a common supplier. We also investigate the conditions where the unique equilibrium exists in the rationing game. Furthermore, this result is also extended to the case where transshipment is allowed between the two retailers.

Among the literature on capacity allocation, the work of Cachon and Lariviere [2,3] is most closely related to our work. Cachon and Lariviere [2] investigate which allocation rules induce retailers to manipulate their orders. They show that proportional and linear allocation rules induce retailers to inflate their orders, whereas the uniform allocation rule induces retailers to order truthfully. They also show that the supplier's performance improves with an allocation rule that induces order manipulation. Cachon and Lariviere [3] address the existence and uniqueness of the equilibrium orders under three allocation rules: (1) the proportional allocation,