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Mathematical models for multicontainer loading problems ☆

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ABSTRACT

This paper deals with the problem of a distribution company that has to serve its customers by putting first the products on pallets and then loading the pallets onto trucks. We approach the problem by developing and solving integer linear models. We start with basic models, that include the essential features of the problem, such as respecting the dimensions of the truck, and not exceeding the total weight capacity and the maximum weigh capacity on each axle. Then, we add progressively new conditions to consider the weight and volume of pallet bases and to include other desirable features for the solutions to be useful in practice, such as the position of the center of gravity and the minimization of the number of pallets.

The models have been tested on a large set of real instances involving up to 46 trucks and kindly provided to us by a distribution company. The results show that in most cases the optimal solution can be obtained in small running times. Moreover, when optimality cannot be proven, the gap is very small, so we obtain high quality solutions for all the instances that we tested.

1. Introduction

Everyday a distribution company has to decide how to put goods onto pallets to serve the customers' orders and then how to efficiently distribute these pallets among the trucks, in order to get the right goods to the right place, and in the desired conditions, while minimizing the number of trucks used. In this paper we will take as a reference a large company in Europe, but the problem is common to many other distribution companies around the world. Solving the problem in an optimal way that produces a reduction in the transportation costs for the companies, thus increasing the profits and also decreasing the greenhouse emissions.

The loading problem consists of two, interrelated, phases. All items have to be placed on pallets; we call this phase *pallet building*, and all pallets have to be placed onto trucks; we call this phase *truck loading*. In the pallet building phase the items are grouped in layers and the layers are stacked on the pallet base. A *layer* is an arrangement of units of the same product, forming a rectangle whose dimensions and number of items are known. A layer completely covers the pallet base in horizontal directions without gaps, ensuring sufficient support for other layers that may be placed on top of it to make up the pallet. The layer composition of each product is decided in advance by the company, so the layer composition is given and we are left with the problem of stacking layers to build pallets. Once the pallets are built, they have to be placed onto the trucks. In the following, we suppose that there is an infinite supply of identical trucks.

The loading problem we deal with is a Multi Container Loading Problem (MCLP). The means of transport, in this case the use of trucks, introduces some constraints that have to be respected for safety reasons. One important feature is the weight limit. There is a maximum weight that can be loaded and this limit cannot be exceeded. There are also limits on the maximum weight each axle can bear. Indeed, there is not only a limit on the total weight of the cargo, but there are also limits on the weight that each axle can bear. Excesses over these weight limits represent a risk for traffic safety and can cause damage to the road. Therefore, they are strictly controlled and the violations severely punished. Some roads have Weight-In-Motion systems, that monitor axle weight violations while driving (see, e.g., Jacob and Feypell-de La Beaumelle [1]). Moreover, for safety reasons the load has to be well spread into the truck to avoid movements during the journey. This means that the center of gravity has to be placed in between the axles and as near as possible to the geometric center of the truck.

Unlike the Single Container Loading Problem (SCLP), that has been extensively studied, the MCLP has attracted less attention so far. In particular, the problem studied here of putting goods on pallets and pallets onto trucks is quite a new issue in the cutting

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and packing literature. As far as we know, just a few previous studies (discussed in the following section) dealt with the MCLP, and not all of them included real constraints such as total weight, axle weight, and position of the center of gravity. Even fewer exact algorithms have been proposed, and just for solving some particular versions of the problem.

For these reasons our proposal is to attempt to solve the problem exactly, in particular by using *Integer Linear Programming* (ILP) models as previously done by Malaguti et al. [2] and Hu et al. [3] who also solve real problems applying ILP models. Apart from being rather easy to implement for a practitioner, ILP models are flexible tools for adding or removing constraints so as to meet the requirements of the specific MCLP at hand, and in recent years have acquired a very good computational behavior, as also witnessed by our results below. We model the characteristics of the problem, starting from a basic model with the main features and then gradually adding new features, one at a time, introducing in the model real constraints required by the company. The models have been tested with real instances provided by a distribution center and the results show that optimal solutions can be obtained in most cases with small computing times. In the cases in which optimality cannot be proven, the solutions usually have a very small gap, and therefore this approach is able to obtain high quality solutions for instances with up to 46 trucks. Our results refer to instances where pallets are loaded onto trucks, but also apply to the loading of containers in general. For that reason in the remaining of the paper we use the terms truck and container as synonyms.

The remainder of the paper is organized as follows. An overview of related existing research is presented in Section 2. In Section 3 the problem is formally described. In Section 4 the test instances are analyzed, and upper and lower bounds for each instance are calculated. In Section 5 we introduce the basic model which includes the main characteristics of the problem, such as not exceeding the truck dimensions and the maximum weight, and then we add axle weight constraints. In Section 6 we include the pallet bases, considering one and two pallets per position. We study additional conditions such as the center of gravity and the minimization of the number of pallets in Section 7. Section 8 contains the conclusions.

2. Previous work

There are not many papers that address the issues studied here, considering pallet and truck loading together. Following the typology for cutting and packing problems introduced by Wäscher et al. [4] the two problems, pallet and truck loading, can be classified as Single Stock Size Cutting Stock Problems. Morabito et al. [5] deal with the same problem but in two dimensions, because the products cannot be stacked. In a first phase, the problem consists in loading the maximum number of products on a pallet. They solve the problem by using the 5-block algorithm proposed by Morabito and Morales [6]. When the pallets are built, they use the same approach to load the pallets onto the trucks. Takahara [7] deals with the problem of loading a set of items on a set containers and pallets. A loading sequence for the items is chosen and it determines the order in which the items are inserted into the bins. The sequence is selected by a metaheuristic method based on a neighbourhood search. A selector determines the sequence of the bins. When a bin is selected, the first item is loaded into the bin, placing it in the first space in which it fits. If the item does not fit into the bin, the next bin is selected. A strategic procedure determines when to exchange the sequence of the items with a neighbour sequence and when the choice of the bin is changed from following the sequence to being randomly chosen, depending on the quality of the solutions. Moura and Bortfeldt [8] deal with the same problem in two steps. In the first step boxes are packed onto pallets, while in the second step these pallets are loaded into trucks. For packing boxes onto pallets they use the method proposed by Moura and Oliveira [9] and they deal with the problem of loading pallets into trucks as a one-dimensional bin packing problem, which is solved by a tree search procedure.

The SCLP, in contrast, is a well-studied problem (see, e.g., Bischoff and Ratcliff [10], Araya and Riff [11], Zhu and Lim [12], Gonçalves and Resende [13,14], Araujo and Armentano [15], Fanslau and Bortfeldt [16], and Junqueira et al. [17]), where the real constraints that we are also facing received an increased attention. According to the survey by Bortfeldt and Wäscher [18], at least 13.9% of the container loading literature deals with weight limit, while weight distribution is considered by 12.1% of the papers. Gehring and Bortfeldt [19], Bortfeldt et al. [20], Terno et al. [21], and Egeblad et al. [22] are some of the authors who include weight limit constraints in their studies. Indeed, when the cargo is heavy, the weight becomes a very restrictive constraint, more than the volume or the space occupied.

Weight distribution constraints require the weight of the cargo to be spread across the container floor, to avoid displacements during the journey or to balance the load between truck axles when the container is transported by truck. To achieve a good weight distribution, the center of gravity of the load should be in the geometrical mid-point of the container floor, as in Bischoff and Marriott [23], or should not exceed a certain distance from it, as in Bortfeldt and Gehring [24] and Gehring and Bortfeldt [19].

Axle weight is a constraint imposed by the means of transport and it has not been widely studied. Lim et al. [25] deal with a particular SCLP with axle weight constraints. They propose an integrated heuristic solution approach that combines a GRASP wall-building algorithm with ILP models. They first apply a customized wall-building heuristic based on the GRASP by Moura and Oliveira [9], including special considerations for box weight and density. Then they use an integrated approach to handle the weight requirements. If the container load limit is exceeded, they unload the necessary number of boxes by iteratively solving an ILP model to meet the requirement. If the axle weight limit is exceeded, they take two steps iteratively until the limit is satisfied: the first step consists in interchanging the positions of the walls created by the customized heuristic, whereas the second step consists in solving an ILP model to unload boxes and in applying one more time the first step to improve the container balance as well as to force a feasible weight distribution.

Haessler and Talbot [26] describe a heuristic for loading customers' orders, and developing load patterns for trucks and rail shipments. The products have low density and for that reason the approach is based on loading by volume rather than by weight. To deal with axle weight constraints, stacks are sequenced by alternating the heaviest and lightest stacks.

Weight constraints also appear in recent studies combining loading and routing, such as, e.g., Iori et al. [27] and Bortfeldt [28]. Doerner et al. [29] deal with a particular vehicle routing problem in which the items are placed on pallets and stacked one above the other, producing piles. They propose two metaheuristic algorithms, a Tabu Search, and an ACO algorithm. A survey on loading and routing problems is presented by Iori and Martello [30]. A recent work not included in the survey is the one by Pollaris et al. [31], that combine a capacitated vehicle routing problem with the loading of homogeneous pallets inside the vehicle. They consider sequence-based pallet loading and axle weight constraints, and propose a mixed ILP formulation for the problem. Pallets may be placed in two rows inside the vehicle but cannot be stacked on top of each other because of their weight, fragility, or customer preferences. Sequence-based loading is assumed to ensure that, when

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