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Review

Unified matrix approach to solve production-maintenance problems on a single machine ☆

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ABSTRACT

We are presenting a general unified matrix framework for production-maintenance systems on a single machine. We have positional processing times and we propose a very general time dependent weighted maintenance system that includes most of the models from the literature. Several performance criteria are included and we show that a broad class of these problems can be modeled as assignment and rectangular assignment problems so that standard software can be utilized to determine their optimal solutions.

Contents

1. Introduction

There is a vast literature on single-machine production systems with learning and deterioration effects. In particular in manual or semi-automatic processes, the processing times may be shortened since the operators can use their past experience. On the other side, the production may slow down with the deterioration of the tools that are to be used on the production site.

In the more recent literature, maintenance periods are to be inserted into the production system to restore the initial processing conditions. Also such maintenance periods may incorporate a learning effect (the maintenance crew may learn from their previous maintenance) or they may deteriorate with time (a postponed maintenance may take longer).

We shall in this paper concentrate on so-called positional changes of the processing times. A feasible schedule is given by the job sequence on a single machine. We have processing times $p(j, r)$

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for job j in position or rank r of the sequence, which may possibly also be influenced by its location with respect to the maintenance intervals.

Frequently, specific analytical functions for the processing times are proposed. We refer as reference on this topic to the recent articles [1–6] with their listed bibliography on production-maintenance systems. For instance in [3,7] general positional processing times are considered without any specific functions.

We also shall adopt this last approach that does not require any particular explicit analytical functions. It is sufficient to capture only the tendencies, i.e., learning or deterioration (Section 2). We define in Section 3 so-called weighted maintenance systems and describe in Section 4 a procedure to set up a matrix that contains all informations about the production-maintenance system for very general processing times and maintenance models. Then the optimal schedules are obtained by solving assignment and rectangular assignment problems and only *a posteriori* the important informations are retrieved, like maintenance durations and completion times. Section 5 treats the simpler makespan criterion and an extension to group dependent models is given in Section 6.

2. Positional production systems

We consider single-machine problems for which the processing time of job j $(j=1,\ldots,n)$ when scheduled in rank r $(r=1,\ldots,n)$ is given by $p(j,r)$. The processing times are *deteriorating* if for all j

$$p(j,1) \le p(j,2) \le \cdots \le p(j,n) \tag{1}$$

and we have a *learning effect* if

$$p(j,1) \ge p(j,2) \ge \cdots \ge p(j,n). \tag{2}$$

Various analytical functions for the processing times $p(j,r)$ have been proposed in the literature. Frequently the processing times are *decomposable*, i.e., they are of the form $p(j,r) = p_j g(r)$ (see [3,8]). Specific functions $g(r)$ include: r^a (see [9–11]) and σ^{r-1} (see [12]). Other functions mentioned are: $p_j + b_j r$ (see [5,13,14]), $p_j r^{a_j}$ (see [4,5,15,16]), $p_j \sigma_j^{r-1}$ (see [17]) and $p_j g_j(r)$ [3]. These processing times are deteriorating if $a, a_j, b_j > 0, \sigma, \sigma_j > 1$ and $g(r), g_j(r)$ are nondecreasing functions in r, whereas we have a learning effect if $a, a_j, b_j < 0, 0 < \sigma, \sigma_j < 1$ and $g(r), g_j(r)$ are nonincreasing.

We shall not assume any analytical form, but rather require that in the given production system one can evaluate in constant time the processing times $p(j,r)$ and hence obtain an $n \times n$-table of values. Since one may have at first a learning effect and then after some time a deterioration because of tool wear, we shall not require any monotonicity of the values $p(j,r)$.

Observation 1. *Let a function $f(j,r)$ be given. Consider the assignment problem based on the nxn matrix $\{f(j,r)\}$. Any feasible solution selects for each row and each column exactly one entry, i.e., each job j is assigned to exactly one rank $r=[j]$ and all ranks r are filled by exactly one job $j=[r]$. Then the optimal solution minimizes*

$$\gamma = \sum_{j=1}^{n} f(j,[j]) \text{ or, equivalently, } \gamma = \sum_{r=1}^{n} f([r],r) \tag{3}$$

and can be obtained in $O(n^3)$ time (see [18]).

Notice that the mappings $j \longrightarrow r = [j]$ and $r \longrightarrow j = [r]$ define permutations, which are inverse to each other.

Now let position-dependent processing times $p(j,r)$ be given. We want to solve one-machine scheduling problems

$1/p(j,r)/\gamma$

for the most common performance criteria γ. We only require that γ can be written in the form (3), where $f(j,r)$ depends on $p(j,r)$. Then we can use Observation 1 to solve the scheduling problem as an assignment problem, which immediately provides the optimal job sequence.

It is difficult to tell who the first was to apply assignment problems to positional production systems. It seems that Lawler made this observation in the 1970s. This result appeared then systematically in the scheduling literature since 1999 [9]. Denote by C_j the completion time of job j in some sequence. It follows a list of some of the most popular criteria γ and the corresponding matrix entries $f(j,r)$ for the assignment problem:

(1) Makespan C_{max}. Here we set simply $f(j,r) = p(j,r)$.
(2) Total completion time $\sum C_j$, where $f(j,r) = (n-r+1)p(j,r)$.
(3) Total absolute difference in completion times, defined as: $TADC = \sum_{i<j} |C_i - C_j|$. Here $f(j,r) = (r-1)(n-r+1)p(j,r)$.
(4) One may take linear combinations of the criteria γ given above:

$$\gamma = \alpha_1 C_{max} + \alpha_2 \sum C_j + \alpha_3 TADC$$

For instance the linear convex combination $w \sum C_j + (1-w)$ $TADC, 0 \le w \le 1$, is proposed in [10]. Then $f(j,r)$ is given by the appropriate linear combination of the functions in (1)–(3).
(5) Further performance criteria from the literature can be obtained with additional parameters. We mention the case that deals with interval constrained processing times $p(j,r)$, initiated recently by Xue et al. [7]. Each job j has a desired processing interval, $p(j,r) \in [a_j, b_j]$. In [7] the porcelain manufacturing process is mentioned, where processing times not within their interval may lead to quality flaws.

We define the processing earliness and tardiness $e_j = \max\{0, a_j - p(j,r)\}$ and $t_j = \max\{0, p(j,r) - b_j\}$ and get performance criteria of the form

$$\gamma + \rho \sum e_j + \sigma \sum t_j,$$

where γ may be any of the criteria (1)–(4). In this case, we obtain $f(j,r)$ by adding to the function in (4) the term $\rho e_j + \sigma t_j$.

In summary, we can combine and generalize many results from the literature as follows.

Proposition 1. *The single-machine scheduling problems*

$1/p(j,r)/\gamma$

for any criterion (1)–(5) can be modeled as assignment problems and solved in $O(n^3)$ time.

It is interesting to mention that for decomposable processing times also the criteria (1)–(4) lead to decomposable matrix entries $f(j,r)$. Thus all these problems can be solved in $O(n \log n)$ instead of $O(n^3)$ by sorting, using the concept of minimal scalar products of two vectors from [19]. Details for the decomposable case can be found in [20,21]. However, criterion (5) will no longer be decomposable for decomposable processing times $p_{jg}(r)$ and also will no longer be deteriorating if the function $g(r)$ is deteriorating with r.

There are also further extensions in the literature, which are not included in this paper, for instance to parallel-machine systems [13,22–26], to time-dependent and mixed models (positional and time-dependent processing times) [23,24,27–33]. In [3,34,35], the concept of so-called group-dependent processing times is considered, which we shall treat in Section 6.

3. Models for maintenance interventions

In an environment with positional processing times $p(j,r)$ that are, at least from some rank on, deteriorating, one has to insert from time to time maintenance periods to return to good processing conditions.

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