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Location choice and risk attitude of a decision maker ☆

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ABSTRACT

In this paper we study the effect of a decision maker's risk attitude on the median and center problems, two well-known location problems, with uncertain demand in the mean–variance framework. We provide a mathematical programming formulation for both problems in the form of quadratic programming and develop solution procedures. In particular, we consider the vertex and absolute median problems separately, and identify a dominant set for the center problem. Glover's linearization method is applied to solve the vertex median problem. We also develop a branch and bound algorithm and a heuristic as the linearization technique takes too long for the vertex median problem on large networks. A computational experiment is conducted to compare the performance of the algorithms. We demonstrate the importance of taking into account the volatility and correlation structure when a location decision is made. The closest assignment property is also discussed for these location problems under the mean–variance objective.

1. Introduction

Decisions to locate facilities such as plants, warehouses and shopping malls are very important, and are often classified as strategic decisions [32,26,16,27]. They usually result in significant fixed costs and more importantly they have considerable impacts on growth prospects of a firm. Moreover, relocating facilities is usually not easy and very costly. As a result, location decisions are made carefully as the executives are aware of their significant economical importance.

To our knowledge, there has been no literature on managerial perceptions of risk specifically for location decision-making problems. However, many studies of risk taking by business executives and managers have attested the importance of risk assessment and management to decision making from the managerial perspective [6]. Most managers interviewed in these studies depicted themselves as risk averse or risk seeking. It has been inferred that their risk attitudes could be attributed to cultural, organizational, occupational and individual differences. Given the substantial impact of a facility location decision, it is arguable that the decision maker may not always be risk neutral, a common assumption in the facility location literature.

1.1. Risk analysis in facility location

It is assumed that the decision maker is risk neutral in all the early and much of the recent facility location literature, in particular, on the median and center problems. However, there are studies that introduce the notion of volatility to these classical location problems. Table 1 summarizes the risk analysis measures used in these studies.

The probability-related measure approach seeks to maximize the probability to achieve a target level of distance or coverage. Value-at-risk (VaR) and conditional value-at-risk (CVaR) are popular measures of risk in finance. β-VaR and β-CVaR at a probability level β are defined, respectively, as the β-quantile of a random loss (or cost) and the conditional expected loss (or cost) exceeding β-VaR [28].

The mean–variance theory [24] is classical in financial portfolio management that makes a trade-off between the mean return and the associated risk. A mean–variance optimization model is to maximize the mean–variance objective function

$$U(Y) = E(Y) - \lambda Var(Y), \quad (1)$$

where Y is a random payoff with mean $E(Y)$ and variance $Var(Y)$, $U(Y)$ is the decision maker's utility, and λ is a risk attitude coefficient. Note that the decision maker is risk averse, risk neutral and risk seeking when $\lambda > 0$, $\lambda = 0$ and $\lambda < 0$, respectively. The magnitude of λ reflects the degree of the decision maker's risk attitude. It was shown that the mean–variance objective is consistent with a quadratic expected utility function [17,30,29,9].

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Table 1
Selected literature on risk analysis in the median and center problems.

Risk measure	Single-facility problems	Multiple-facility problems
Probability measure	Frank [10,11]	
	Berman et al. [3]	
Variance	Frank [10]	
Mean–variance		Jucker and Carlson [20]
		Hodder and Jucker [18]
Value-at-risk	Wang [34,35]	Wagner et al. [33]

The mean–variance approach has been criticized for taking into account both the favorable and unfavorable deviations of the random payoff Y from the mean $E(Y)$ in its risk measure, namely the variance $Var(Y)$. As punishing desirable fluctuations when the probability distribution of Y is asymmetric may lower the mean payoff, alternative measures that consider the downside risk only have been proposed [25]. However, the studies by Grootveld and Hallerbach [15], and Choi and Chiu [8] suggested that the mean–variance approach and the mean-downside-risk approaches tend to return similar results in most cases. Grootveld and Hallerbach [15] also pointed out that downside-risk measures were much more prone to estimation risk than the variance.

Knowing the controversy over the mean–variance approach, we nonetheless adopt Eq. (1) as the optimization criterion in the current study for the reasons stated below:

- The mean–variance approach is applicable to explore the trade-off between the mean and variance of the random payoff Y for any stochastic optimization problem, including the location analysis problem.
- Using variance to quantify risk is intuitive, the mean–variance model can be applied by decision makers of different risk attitudes (risk neutral, risk averse and risk seeking), and the mean–variance approach requires only the first two moments of the random variable Y. On the contrary, alternative risk measures such as VaR and CVaR usually reflect the downside risk-averse behavior only, and entail the knowledge of the probability distribution of the random payoff Y.
- Unlike the mean–variance optimization model with a constraint to bound variance from above, the mean–variance objective (Eq. (1)) does not exclude solutions with a high mean payoff and high variability from consideration. Therefore, we would not expect that the negative impact the approach's flaw has on solution quality be significant as long as the risk attitude coefficient λ is not too large.

As a remark, we realize that there are controversies over the pros and cons of various risk measures and believe that a comparative study on these measures for location problems would greatly facilitate the application of stochastic location analysis models.

The mean–variance objective was used by Jucker and Carlson [20] and Hodder and Jucker [18] to study the uncapacitated plant-location problem with uncertain prices and demand. Hodder and Jucker's problem is to some extent similar to the version of the p-median problem to be studied in this paper. A major difference is that they chose a very specific correlation structure for the prices charged by facilities (which were the random variables in their model) whereas we use a general correlation structure for the random demand weights. Consequently, our optimization problem is much harder than theirs.

In Wagner et al. [33], the uncapacitated plant-location problem was studied with an objective to maximize the VaR of a future profit. Under the normality assumption the objective function was converted into the mean–variance framework and a nonlinear integer programming model was solved. The algorithmic approach proposed by Wagner et al. [33] works only for a risk-averse decision maker. The authors reported a computational experiment on small networks only for the vertex version of the model, where facilities can be located on the nodal points only. Different from their study, in this paper we discuss both the absolute (facilities can be located anywhere on the network) and vertex p-median problems, and conduct an extensive computational study on networks of various sizes. We also develop a motivating example to show that when the mean value of Y is non-decreasing in the variance the decision maker may have a reason to be risk seeking, and present algorithms that are not restricted by the decision maker's risk attitude.

1.2. Outline of the study

In this paper we consider the decision maker's risk attitude and investigate how it can change the optimal solutions to the median and center location problems with uncertain demand weights that were well studied under the assumption of risk-neutrality. Our main objective is to show how the decision maker's risk attitude can affect the optimal facility locations. We also try to shed some light on the important role that demand variability and correlation structure play to choose optimal risk-averse or risk-seeking locations.

Under the mean–variance objective, each optimization problem is formulated in the form of a quadratic programming model. We discuss how we can solve the problems using linearization techniques, which have been shown to be quite effective for problems with quadratic objective function [7]. Specifically, Glover's [14] linearization method is adopted for the vertex median problem. We also develop a branch and bound algorithm and a vertex substitution heuristic for the problem because our computational experience suggests that the linearization method is not always efficient for large networks.

The rest of the paper is organized as follows. In Section 2, we discuss the mean–variance objective, and in Sections 3 and 4 we analyze the median and center problems with uncertain demand under the mean–variance objective. In Section 5, we provide insights on optimal locations under different risk preferences and changing parameters. In Section 6, a computational experiment is reported to compare the performance of the algorithms developed for the models. Finally, we provide a brief summary and outlook in Section 7.

2. The mean–variance objective

Let $G=(N,L)$ be an undirected network with a set of nodes N ($|N|=n$) and a set of links L. Let x denote both the location of point x on link (a,b) and the distance between this point and the left end node a. The shortest distance between any two points x and y located somewhere in G is denoted by $d(x,y)=d(y,x)$. To uniquely define a link $(a,b)\in L$, it is required that the index of the left end node a is smaller than that of the right end node b. We further require that the length of each link (a,b), denoted by l_{ab}, is equal to the shortest distance between nodes a and b.

Given a set of p ($p<n$) points $\mathbf{X}_p=(x_{(1)},x_{(2)},\cdots,x_{(p)})$, let $d(x,\mathbf{X}_p)=\min_{1\le j\le p}\{d(x,x_{(j)})\}$. Demand is assumed to originate from the nodes of G only. The demand weight at node i, h_i, is random with mean μ_i and standard deviation σ_i. Random variables h_i and h_k may be correlated with a correlation coefficient ρ_{ik}.

We study the median and center problems with random demand weights under the mean–variance objective (Eq. (1)), for which the random payoff Y will be, respectively, defined for either

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