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A posteriori evaluation of simplification details for finite element model preparation

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ABSTRACT

To perform a mechanical simulation of a component, its geometrical model usually needs to be simplified in accordance with the hypotheses on its mechanical behaviour. Here, the preparation and the simplification of the model for the structural analysis act upon an intermediate polyhedral representation and take into account the mechanical hypotheses specified on the product shape. An a posteriori mechanical criterion has been incorporated into this process to bring an objective estimation of the model simplification. The a posteriori criterion can be applied to FE problems of linear static analysis or thermal problems for stationary linear conduction and is able to estimate the influence of shape transformations over the global analysis results. If a shape detail removed during the shape simplification process proves to be influent on the mechanical behaviour, it can be re-inserted on the simplified model, so readapting the initial simulation model. In this article, the focus is on the description of the geometric operators supporting the automatic process of FE model preparation.

1. Introduction

During the product development process, the simulation stage, commonly named "engineering analysis", evaluates the physical behaviour of a product component subjected to various kinds of loads and boundary conditions. Finite element (FE) approaches are techniques widespread in industry to analyse the mechanical behaviour of a component. A FE analysis gives useful information about the component's mechanical behaviour, helpful for ensuring that design requirements are satisfied. The meaningfulness and accuracy of a FE computation are the main user's concerns. Among the possible sources of error that could affect the quality of the FE results, we can mention inappropriate domain discretization and/or uncertainty about the boundary conditions [1–4].

Yet, the transformations performed on the component shape are a critical element to take in account, and their choice and control are of essential importance. Indeed, models generated during the design process need to be prepared before performing a mechanical simulation. Using CAD models, resulting from a design process and dedicated to manufacturing purposes, could introduce some problems of FE mesh generation and solving. Therefore, the generation of models for structural behaviour simulation needs several steps of shape adaptation and idealization [5–7], where shape sub-domains having negligible influence on the FE analysis are removed. The choice and evaluation of the corresponding shape simplifications to perform is of primary importance.

Many approaches were developed on this topic, centred on some a priori criteria. These criteria act before performing a FE analysis, and drive and control the shape changes that occur on the component. They can be either subjective, i.e. based on the user's expertise [8,9], or objective, i.e. based on geometrical criteria linked to some mechanical properties of the problem (like variation of volume and mass) [10,11]. Nevertheless, a priori criteria are not able to quantify accurately the real influence of a shape simplification on some parameters of the FE simulation output. In fact, they cannot refer to quantities obtained from the simulation results, like displacement and stress fields. Therefore, a more precise mechanical criterion needs an a posteriori approach. In such a case, this criterion is applied after performing a FE simulation on a simplified model, and is targeted on some objective parameters, like stresses, strains or strain energy. Although some a posteriori approaches exist [12], no much work has been dedicated to this topic yet.

We developed an a posteriori criterion [13] that can be applied to FE problems of linear static analysis or thermal problems for stationary linear conduction. It uses an approximation of the energy norm of the difference between the FE solution on the initial and simplified model, and it is able to evaluate the influence on global simulation results caused by shape details removal. The use of an a posteriori FE error estimator can be incorporated in an adaptive process of geometric simplification [14]. In fact, the shape of the simplified part could be adapted after a first simulation, depending on the influence of its removed details on the FE results.

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The paper is an updated and revised version of the conference paper [15], and is structured as follows. Section 2 explains the principles about the developed a posteriori influence indicator and Section 3 describes the overall scheme of the adaptive simplification process where this indicator is employed. Section 4 introduces the concept of "simplification detail" and some of the corresponding shape modification operators. Finally, Section 5 illustrates how to prepare the FE problem for applying the a posteriori indicator, and how to interpret indicator's results.

2. A posteriori error indicator

The developed indicator can be applied in FE static computation of linear elastic structures. As shown in Fig. 1, two categories of shape changes are taken into account according to the geometric domain variations:

– Addition of the sub-domain Γ_A.
– Subtraction of the sub-domain Γ_S.

2.1. Influence of a shape transformation

Using the example shown in Fig. 1, here we briefly explain how the a posteriori indicator is computed [13]. Let us assume that the solution of the initial FE problem over the domain Ω_1 returns the displacement field $\vec{U}_1$, the stress field $\bar{\bar{\sigma}}_1$ and the strain field $\bar{\bar{\varepsilon}}_1$. We call $\partial\Omega_1$ the boundary of Ω_1. In the same way, $\vec{U}_2$, $\bar{\bar{\sigma}}_2$ and $\bar{\bar{\varepsilon}}_2$ are the solution fields of the simplified FE problem on the simplified domain Ω_2, having $\partial\Omega_2$ as boundary.

We can assume that the simplified problem matches exactly the first one, i.e. the error vanishes, if:

– On the intersection of the two domains, i.e. $(\Omega_1 \cap \Omega_2)$, the initial and simplified problem solutions are equal.
– Over the sub-domain Γ_A, the stress and strain fields, $\bar{\bar{\sigma}}_2$ and $\bar{\bar{\varepsilon}}_2$ respectively, are equal to zero.
– Over the sub-domain Γ_S, the stress and strain fields, $\bar{\bar{\sigma}}_1$ and $\bar{\bar{\varepsilon}}_1$ respectively, are equal to zero.

To estimate the influence of these shape modifications, we need to assess:

– the difference $(\vec{U}_1 - \vec{U}_2)$ over the common sub-domain $(\Omega_1 \cap \Omega_2)$;
– the stress field $\bar{\bar{\sigma}}_2$ over Γ_A;
– the stress field $\bar{\bar{\sigma}}_1$ over Γ_S.

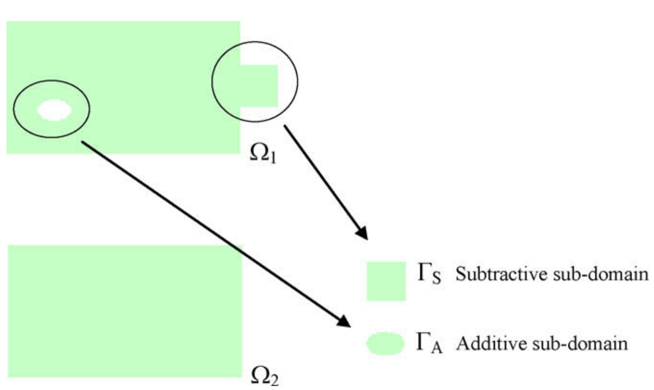

Fig. 1. Simplification example: an initial domain Ω_1 and the corresponding simplified domain Ω_2. To produce Ω_2 the sub-domain Γ_A is added and the sub-domain Γ_S is subtracted from Ω_1.

We used an energy norm to measure these quantities. The corresponding error, e, is given by

$$2e^2 = \int_{\Omega_1\cap\Omega_2} (\bar{\bar{\sigma}}_1 - \bar{\bar{\sigma}}_2) : (\bar{\bar{\varepsilon}}_1 - \bar{\bar{\varepsilon}}_2)\mathrm{d}\Omega + \int_{\Gamma_A} \bar{\bar{\sigma}}_2 : \bar{\bar{\varepsilon}}_2\,\mathrm{d}\Omega + \int_{\Gamma_S} \bar{\bar{\sigma}}_1 : \bar{\bar{\varepsilon}}_1\,\mathrm{d}\Omega. \tag{1}$$

In the case where the boundaries of the simplified sub-domains Γ_A and Γ_S are free, i.e. null Neumann conditions are applied, we can simplify Eq. (1) and obtain Eq. (2). Here, $\vec{n}_\Omega$ is the normal vector pointing outward from the domain Ω, and $\vec{f}d$ designates the volumetric field of forces applied to Ω. Then

$$2e^2 = \int_{\Gamma_A} \vec{f}d\cdot\vec{U}_2\,\mathrm{d}\Omega + \int_{\Gamma_S} \vec{f}d\cdot\vec{U}_1\,\mathrm{d}\Omega + \int_{\partial\Gamma_A\cap\partial\Omega_1} \left[\bar{\bar{\sigma}}_2\cdot\vec{n}_{\Gamma_A}\right]\cdot\vec{U}_1\,\mathrm{d}\partial\Omega + \int_{\partial\Gamma_S\cap\partial\Omega_2} \left[\bar{\bar{\sigma}}_1\cdot\vec{n}_{\Gamma_S}\right]\cdot\vec{U}_2\,\mathrm{d}\partial\Omega. \tag{2}$$

The error e is an absolute error. A more meaningful relative error, η, can be expressed in terms of the strain energy of the problem, as

$$\eta^2 = \frac{e^2}{\frac{1}{2}\int_{\Omega_1}\mathrm{Tr}\left[\bar{\bar{\sigma}}_1\bar{\bar{\varepsilon}}_1\right]\mathrm{d}\Omega} \approx \frac{e^2}{\frac{1}{2}\int_{\Omega_2}\mathrm{Tr}\left[\bar{\bar{\sigma}}_2\bar{\bar{\varepsilon}}_2\right]\mathrm{d}\Omega}. \tag{3}$$

The computation of the error e with Eq. (2) would need to know the solution on the initial domain Ω_1 (quantities with subscript 1). Since our aim is to avoid solving the initial FE problem, this initial solution is unknown. Therefore, we estimate it by using a local FE computation over a sub-domain surrounding each suppressed detail, that we name Ω_{2S} or Ω_{2A}, depending on whether it surrounds a subtractive or an additive sub-domain. Fig. 2 shows an example of such sub-domains' surrounding, with reference to the sub-domains removed in Fig. 1. According to the removed sub-domain's type we have $\Delta_S = \Gamma_S \cup \Omega_{2S}$ or $\Delta_A = \Gamma_A \cup \Omega_{2A}$.

If we address shape simplifications of subtractive type, we have a reduction of the initial model Ω_1 following the subtraction of the sub-domain Γ_S, where Γ_S and the neighbouring sub-domain Ω_{2S} are adjacent. The stiffness of Δ_S is computed as the sum of the stiffnesses of the FE meshes of Γ_S and Ω_{2S}. In contrast, the case of shape simplifications of additive type, implies an increase of the initial model Ω_1 due to the addition of a sub-domain Γ_A. Γ_A is completely immersed in its neighbouring sub-domain Ω_{2A} and the stiffness of Δ_A is computed as the difference between the stiffnesses of the FE meshes of Ω_{2A} and Γ_A.

The boundary conditions of the FE local problems on Δ_S and Δ_A are given by the displacement field $\vec{U}_2$, which results from the FE computation over the simplified problem Ω_2. Bold lines in Fig. 2 correspond to the boundaries where displacements from the field $\vec{U}_2$ are applied.

Local FE computations allow us to give an estimation of the relative error, e_{est}, as

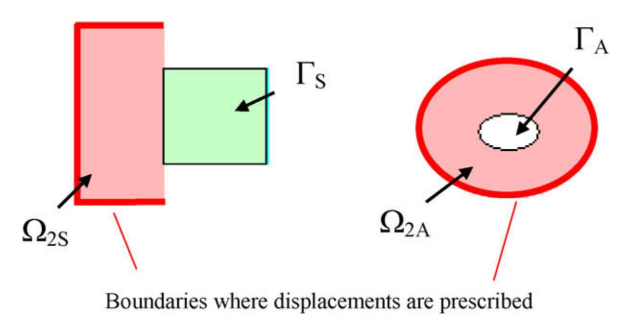

Fig. 2. Neighbouring sub-domains, Ω_{2S} and Ω_{2A}, for the FE local computations around Γ_S and Γ_A, respectively, related to the example of Fig. 1.