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Topology design of two-dimensional continuum structures using isolines

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ABSTRACT

This paper presents the algorithm for the topological design of two-dimensional structures using isolines called isolines topology design (ITD). The topology and the shape of the design depend on an iterative algorithm, which continually adds and removes material depending on the shape and distribution of the contour isolines of the required structural behaviour. In this study the von Mises stress was investigated. Several classic examples are presented to show the effectiveness of the algorithm, which provides quality solutions with very detailed contour without the need to interpret the topology in order to obtain a final design.

1. Introduction

Layout optimisation was pioneered by Michell [1], who studied statically determinate trusses for a number of loading and support conditions. This class of structures can have up to an infinite number of members of varying length and constant cross-section, making them impractical in engineering applications. However they are ideal in providing analytical optimal topologies which can be used to benchmark numerical optimisation methods.

In the 1960s, layout optimisation was significantly improved when the ground structure approach was first introduced [2]. Originally, ground structure problems were solved using direct optimisation methods, e.g. the mathematical programming (MP) algorithms. However, such algorithms are inefficient in solving large optimisation problems. Instead, indirect optimisation (e.g. optimality criteria (OC) algorithms), can be used to solve realistic problems with a large design domain and number of design variables. The criterion may be related to the structural stresses, where a fully stressed design (FSD) of minimum weight is sought [3]. Compared with the MP algorithms, the OC methods are efficient in large optimisation, but lack generality in different kinds of optimisation problems.

Topology optimisation methods can be divided into one of two types: (1) those with a mathematical basis (homogenisation and SIMP) which optimise global criteria (such as compliance) but where the results are dependent on the size of the finite element mesh [4]; and (2) those based on heuristics, (soft kill (SK), hard kill (HK), evolutionary structural optimisation (ESO), additive evolutionary structural optimisation (AESO), bidirectional ESO (BESO), reverse adaptivity (RA), etc.) which optimise local criteria (such as stress) at the finite element (FE) level. Both types primarily deal with the properties at the element level rather than of the structure as a whole.

In the homogenisation method [5], the design domain is divided into a finite number of cells. Where each cell can have its own individual microstructure. The drawbacks of this method are that: (a) it may converge to a local optimum and (b) a post-processing penalization step is necessary to transform the individual microstructure into a continuous solid solution (which may not be physically meaningful).

The solid isotropic microstructures with penalty (SIMP) [6–8] method is considered as a generalization of the variable thickness sheet problem. The material covers the complete range of density values from 0 to 1, but does not provide regularization. This problem disappears if a penalty is included in the formulation. A slight drawback of the SIMP method is that the topology obtained is somewhat dependent on the power law. However its strength is that it can deal with multiple objectives and problems which are multidisciplinary [9]. Some work has also been done to deal with stress constraints which is a local effect, but this was dealt with as a constraint rather than as the optimality criteria [10].

The heuristic methods of topology design primarily rely on producing a fully stressed design by removing a small quantity of

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material in the regions of the design domain where the driving criteria (such as the von Mises stress) have a low value. Several such methods have been developed: SK [11], HK [12], ESO [13–15], RA [16], metamorphic development (MD) [17].

From an engineer perspective, the heuristic methods have some attractive features, they are: (a) relatively simple to program using any Finite Element Analysis (FEA) packages, (b) easy to understand how they work, and (c) the resulting topologies are similar to analytical results (e.g. Michell trusses). On the other hand, the fundamental drawbacks of these methods are: (a) relatively high solution time; (b) strong dependence of the solution on the mesh element size; (c) only some methods can add or reinstate elements into the design domain [17,18], (d) in most of these methods the boundaries are represented by the jagged-edges of the finite element, which require smoothing or image filtering to manufacture a smooth topology. RA [16], MD [17], fixed-grid ESO (FG-ESO) [19], and ESO with boundary element method (BEM) [20] are able to produce smoother boundaries.

This paper presents the implementation of topology design using the isolines of the desired structural performance. The novelty of this work is that the use of isolines guarantees that whilst optimising a local effect (such as stress), the design is carried out at the global-structural level rather than at the elemental level. The method of determining the isolines within a fixed-grid FE is given, together with a brief explanation of the FG-FEA method. Several classic examples are presented to show the effectiveness of the algorithm. The results show the effectiveness of the algorithm, providing quality solutions with very detailed contours, without the need to interpret the topology in order to obtain a final design.

2. Fixed grid finite element analysis

Finite element analysis has played a primary role in the development of computer aided design (CAD). FEA procedures are used in the design of buildings, electric motors, airframes, heat engines, ships, etc. A modern finite element program consists of a graphic modeller, a preprocessor, a finite element solver, and a postprocessor. Traditionally, geometric modelling and FEA are two separates processes. Due to the complexity of some structures/designs, the development of a finite element model of such a structure in some cases can take longer than the actual solution time of the finite element equations. Two reasons for this are: (1) Preprocessing is not a fully automatic process requiring a high level of user–computer interaction, and (2) automatic mesh generators may use algorithms with low performance.

Since at the early stages of a design process the topology of a structure is not fully developed, a fast means of estimating the design criteria with an acceptable level of error in the estimation is required. The fixed-grid FEA (FG-FEA) method has previously been used in problems where either the geometry or physical property of a structure change with time [21]. In this work the fixed-grid method is used to calculate the optimisation driving criteria for the structure.

In FG-FEA, the elements are in a fixed position in space (Fig. 1) and have the real structure superimposed on them. This means that there are elements which lie inside (I), outside (O), or on the boundary (B) of the structure [21,22].

Owing to the fixed-grid geometry of all the finite elements, the stiffness matrix for each element is almost fixed and only depends on its material properties (elasticity module). For the case of inside and outside elements these properties are constant. For boundary elements, the properties consist of a combination of the I and O materials. The fixed-grid approximation then transforms the bi-material element into a homogeneous isotropic element where

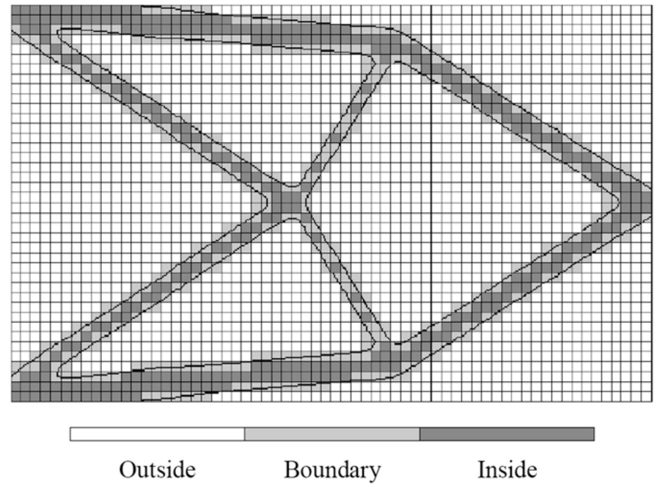

Fig. 1. Fixed grid approximation of the structure. Classification of the finite elements according to their position with regard to the isolines.

the material property is scaled by the function (1) of the area of material I within that element

$$\xi^{(e)} = \frac{A_I^{(e)}}{A_I^{(e)} + A_O^{(e)}} = \frac{A_I^{(e)}}{A_t^{(e)}}, \tag{1}$$

where $A_I^{(e)}$ and $A_O^{(e)}$ represent the area inside and outside the element, respectively, and $A_t^{(e)}$ is the total element area.

If the fixed-grid domain is divided into equal size elements, the stiffness matrix entries are linearly proportional to their normalized fraction area. This greatly reduces the time taken in generating the stiffness matrix every time the boundary changes.

The elemental stiffness matrix is given by (2)

$$\mathbf{K}^{(e)} = \left\{ \begin{array}{ll} \mathbf{K}_I & \text{if } \xi^{(e)} = 1, \\ \mathbf{K}_O & \text{if } \xi^{(e)} = 0, \\ \mathbf{K}_B = \mathbf{K}_I \xi^{(e)} + (1 - \xi^{(e)})\mathbf{K}_O & \text{if } 0 < \xi^{(e)} < 1, \end{array} \right\} \tag{2}$$

where $\mathbf{K}_O$ is the element stiffness matrix for an element outside, $\mathbf{K}_I$ element stiffness matrix for an element inside, and $\mathbf{K}_B$ for an element boundary. Normally $\mathbf{K}_O < 10^{-4} \times \mathbf{K}_I$.

The value of the criteria in each node of an element $\sigma^{(n)}$ is calculated using (3)

$$\sigma_i^{(n)} = \frac{1}{N} \sum_{e=1}^{N} \sigma_e^{(n)}, \tag{3}$$

where $\sigma_e^{(n)}$ is the nodal values of the criteria at node i for each element surrounding that node. Where the nodal value is determined from the criteria values at each Gauss point extrapolated to the nodes using the shape functions of the element, and N is the number of elements connected to that node.

If there is a non-design region, the material properties of the fixed grid elements are not altered during the design process.

3. Optimization with isolines

In [23], topology optimisation is taken as being an extension of shape optimisation. Where firstly the topology of the structure is optimised followed by shape optimisation. In another study [24], the opposite was done, where first the shape was optimised followed by topology optimisation (using homogenisation) to determine the material orientation. Irrespective of the order in which the optimisation was carried out (shape first, topology second, or vice versa), the shape is dependent on the material distribution, and equally the material distribution is dependent on the shape.