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Modelling cohesive crack growth in concrete beams using scaled boundary finite element method based on super-element remeshing technique

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ABSTRACT

A super-element remeshing technique is developed to model cohesive crack growth based on the linear asymptotic superposition assumption. The remeshing operation only occurs along the crack path. Mesh size is refined merely at the crack-tip super-element. The stress intensity factors are solved semi-analytically by the scaled boundary finite element method, sufficient accuracy can be ensured. The cohesive tractions are treated as side-face forces, and the induced displacement field can be sought as a particular solution to the governing differential equations. Numerical examples validate the efficiency and accuracy of the proposed approach.

1. Introduction

In recent years, considerable research effort has been devoted to model crack development in structures made of quasi-brittle materials such as concrete. Discrete crack models based on either the linear elastic fracture mechanics (LEFM) or nonlinear fracture mechanics (NFM) have been employed to predict crack propagation in concrete structures [1–9]. Whether the LEFM- or NFM-based models apply depends on the size of the fracture process zone (FPZ) relative to the structural dimensions. FEM-and BEM-based discrete crack models are two most common models in fracture analyses for their respective unique features. In order to evaluate the fracture parameters (e.g. singular stresses and stress intensity factors (SIFs) etc.) accurately, FEM-based models need enriching crack-tip meshes or introducing singular crack-tip element, which unduly exacerbate the complexity in remeshing algorithm. The BEM-based models avoid a large part of the remeshing because only the boundary of the domain needs to be discretized, it has gained considerable success in crack analysis [10–15]. However, BEM requires fundamental solutions and the solutions are rather complicated, these disadvantages may weaken its merits.

The extended finite element method (XFEM) has shown enormous potential in modelling cohesive crack propagation without remeshing [16]. Nevertheless, in case the crack propagation path is a priori unknown, a dense initial mesh must also be required to predict crack propagation path with sufficient accuracy in the high stress-singular regions. Meshfree or meshless methods [17–22] are attractive in some cases.

The scaled boundary finite element method (SBFEM) developed by Wolf and Song [23–27] is a new semi-analytical approach combining more than the advantages of FEM and BEM. The modelled spatial dimension is reduced by one, only the boundary of the domain and the common edges of super-elements are discretized. Difficulties arising from singularities at the crack-tip can be circumvented because stresses are solved analytically in the radial direction. This enables the SIFs directly extracted from the stress solutions at nodes on the domain boundary or common edges of super-elements without further enriching crack-tip meshes or introducing singular elements [28–30]. Yang and Deeks [31,32] proposed a two-step FEM–SBFEM coupled method to model cohesive crack propagation. They use LEFM to predict crack paths and incorporate cohesive interface finite element (CIE) to take into account the cohesive tractions in the FPZ. Ooi and Yang [33] extended it to deal with multiple crack propagation in concrete. However, inserting CIE into the crack paths after remeshing faces some difficulties, the authors proposed a "shadow domain" method to ease the treatment. In addition, the core subdomains may become so distorted in case of complex crack path and there is difficulty to

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place the scaling centre such that the full boundary can be visible from it. Later, Ooi and Yang further improved their approach with a novel hybrid FEM–SBFEM method [34,35].

In this study a remeshing technique based on SBFEM is proposed, which has the advantages that the mesh layout of super-elements in the domain can be done in such a way to best fit the crack growing path and the density of discretization nodes along the sides of super-elements are specified to meet the desired accuracy of the results. A linear asymptotic superposition assumption [36] is made to simplify the computation, the cohesive tractions to model energy dissipation in the FPZ are treated as side-face forces and the induced displacement field and stress field can be solved analytically without additional effort such as inserting the CIEs in the crack path. As a result, the whole problem can be handled as a linear elastic one. Only the linear elastic fracture mechanics (LEFM) criteria is needed to predict the crack trajectory. And the crack growth is judged from vanishing of the mode-I SIF K_I in the crack propagation direction (see Moes and Belytschko [16] and Yang and Deeks [31,32]).

The contents of this paper are organised as follows. The fundamentals of the SBFEM and the technique of extracting SIFs based on the theory of LEFM are firstly presented and followed by the super-element remeshing technique. Then the concept of the linear asymptotic superposition assumption and the algorithm to calculate K_I with the cohesive tractions in the FPZ are addressed. Finally, a single-notched three-point bending beam and a single notched four-point shear beam are analysed using the proposed method, and the results are compared with experimental data and those available in the literature, fairly good agreement is reached.

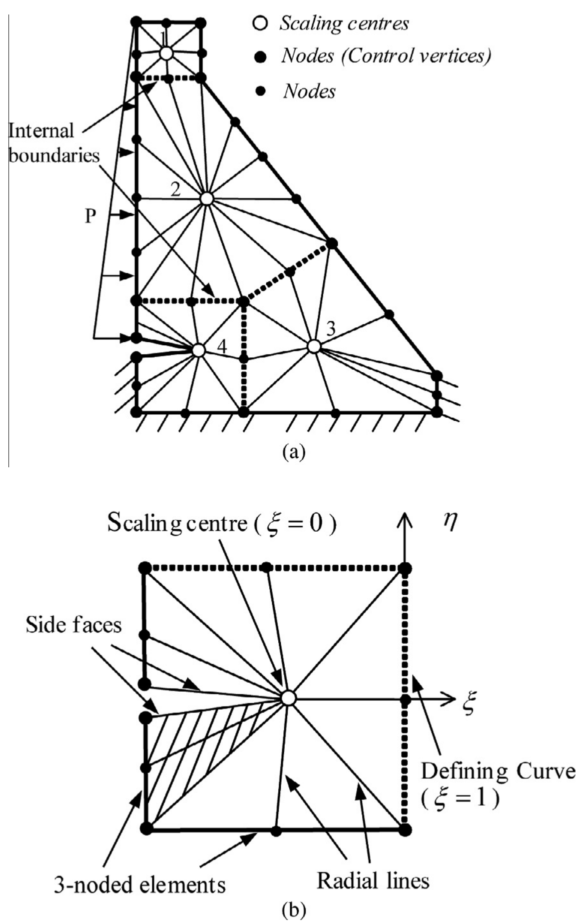

Fig. 1. The concept of the scaled boundary finite-element method: (a) subdomaining of a domain, (b) super-element 4.

2. Basic theory of SBFEM and calculation of SIFs for traction-free cracks using SBFEM

The fundamentals of the SBFEM may be found in the publications [24,26–30,37] and only some key equations for the developments in this paper are summarized.

As shown in Fig. 1a, the domain is conveniently divided into a few super-elements whose size and shape can be arbitrary, and only the visibility from the scaling centre is considered. A typical super-element is depicted in Fig. 1b, only the boundary is discretised. The SBFEM coordinates are defined as ξ and η, where dimensionless radial coordinate ξ pointing from the scaling centre O to a point on the boundary, with $\xi = 0$ at O and $\xi = 1$ on the boundary. η is running in the circumferential direction parallel to the boundary.

The governing equations derived using Galerkin's weighted residual method [38] are expressed as follows

$$\left[E^0\right]\xi^2\{u(\xi)\}_{,\xi\xi} + \left(\left[E^0\right] - \left[E^1\right] + \left[E^1\right]^{\mathrm{T}}\right)\xi\{u(\xi)\}_{,\xi} - \left[E^2\right]\{u(\xi)\} + \xi\{F_{\mathrm{t}}(\xi)\} = 0 \tag{1}$$

where coefficient matrices $[E^0]$, $[E^1]$ and $[E^2]$ depend on the geometry and material properties of the elements, they are independent of ξ, discretization only on the boundary is needed. $\{F_t(\xi)\}$ denotes the surface tractions or loads on the side-faces (or crack faces, see OB, OC in Fig. 2). $\{u(\xi)\}$ represents the nodal displacements.

For homogeneous case without loads on the side-faces $\{F_t(\xi)\} = 0$, Eq. (1) is transformed into the first order ordinary differential equation

$$\xi\{X(\xi)\}_{,\xi} = -[Z]\{X(\xi)\} \tag{2}$$

with

$$[Z] = \begin{bmatrix} \left[E^0\right]^{-1}\left[E^1\right]^{\mathrm{T}} & -\left[E^0\right]^{-1} \\ -\left[E^2\right] + \left[E^1\right]\left[E^0\right]^{-1}\left[E^1\right]^{\mathrm{T}} & -\left[E^1\right]\left[E^0\right]^{-1} \end{bmatrix},$$

$$\{X(\xi)\} = \begin{Bmatrix} \{u(\xi)\} \\ \{q(\xi)\} \end{Bmatrix}$$

$$\{q(\xi)\} = [E^0]\xi\{u(\xi)\}_{,\xi} + [E^1]^{\mathrm{T}}\{u(\xi)\} \tag{3}$$

$\{q(\xi)\}$ is the internal nodal forces corresponding to nodal displacements $\{u(\xi)\}$.

Solve the standard eigenvalue problem

$$[Z][\Phi] = -[\Phi][\Lambda] \tag{4}$$

$[\Lambda]$ and $[\Phi]$ are partitioned into

$$[\Lambda] = \begin{bmatrix} -[\lambda_i] & \\ & [\lambda_i] \end{bmatrix}, \quad [\Phi] = \begin{bmatrix} [\Phi_{11}] & [\Phi_{12}] \\ [\Phi_{21}] & [\Phi_{22}] \end{bmatrix}$$

where eigenvalues appear in pairs and real components of $[\lambda_i]$ are positive.

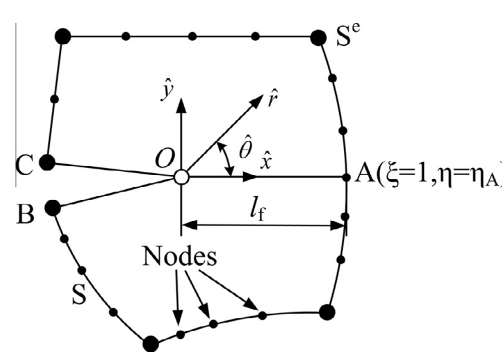

Fig. 2. A cracked domain modelled by super-element with scaling centre at the crack tip.