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Cylindrical bending of piezoelectric laminates with a higher order shear and normal deformation theory

Tarun Kant *, S.M. Shiyekar

Department of Civil Engineering, Indian Institute of Technology Bombay, Powai, Mumbai 400 076, India

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Abstract

An analytical solution, based on a higher order shear and normal deformation theory, is presented for the cylindrical flexure of piezoelectric plates. The primary displacement terms are expanded in thickness coordinate and an exact nature of electric potential is obtained in actuator and sensing layers. The electric potential function is evaluated by solving a second order ordinary differential equation satisfying electric boundary conditions along thickness direction of piezoelectric layer. A unidirectional composite plate attached with distributed actuator and sensor layers is analyzed under electrical and mechanical loading conditions and comparison of results with exact solution is presented. Results for non-piezoelectric plates are also compared with elasticity and other solutions of cylindrical bending.

Keywords: Higher order theories; Piezoelectric plates; Cylindrical bending; Plane strain

1. Introduction

Materials with property to change shape and size when electrically charged and the reversal behavior is utilized in the controlling mechanism of the structures. Piezoelectric layers are embedded or attached to the elastic layers in patches or in a distributed form. Such structures are called as smart/intelligent or adaptive structures.

Tiersten [1] defined material constitutive relations of linear piezoelectricity. The equations of linear piezoelectricity are coupled with the charge equation of electrostatics by means of piezoelectric constants. Earlier Mindlin [2] presented approximate theory for the vibrations of piezoelectric plates.

Ray et al. [3] developed exact solutions for a mono-layered piezoelectric polymer polyvinyledene fluoride (PVDF) plate, under electric potential and mechanical loading. Numerical results are evaluated for thick and thin single piezoelectric layer. Ray et al. [4] further established elasticity solutions for smart unidirectional composite plates under cylindrical bending. Heyliger and Brooks [5] also presented exact solutions of plates with two different layers of piezoceramics, two layers of angle-ply piezopolymers and three layers of cross ply piezopolymers under cylindrical bending. Later Saravanos and Heyliger [6] presented a classified review of the analytical solutions presented by various investigators in the mechanics of the laminated piezoelectric structures. Exact plane strain solution for a piezoelectric orthotropic flat panel under mechanical, thermal, and electric loading is obtained by Dube et al. [7]. Shang et al. [8] also obtained exact plane strain solution for piezoelectric layers under thermal excitation. Dumir et al. [9] presented first order Reissner and Mindlin [10,11] plate (FOST) and classical Kirchhoff plate (CPT) solutions for hybrid plates in cylindrical bending under thermoelectric loading. Vel and Batra [12] used Eshelby–Stroh formulation to analyze cylindrical bending of laminated composite plate with segmented actuators and sensors for different boundary conditions under dynamic state. Static and dynamic response of adaptive angle-ply

* Corresponding author. Tel.: +91 22 2576 7310; fax: +91 22 2576 7302.
E-mail address: tkant@civil.iitb.ac.in (T. Kant).

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laminates in cylindrical bending is studied by Chen et al. [13] using state space approach. Saravanos [14] discussed finite element (FE) formulation based on mixed theory. In this theory, elastic displacements are modeled by equivalent single layer (ESL) theory and electric potential is by layerwise (LW) approach. Similar approach is proposed by Ballhause et al. [15] who have presented statics and dynamics of piezoelectric plates. Piezoelectric plate elements based on Reissner–Mindlin assumptions is presented by Kogl and Bucalem [16] and Carrera [17]. Mannini and Gaudenzi [18] investigated a stress concentration problem in the smart composite using higher order FE model. An iterative FE solution is presented by Gaudenzi and Bathe [19] and is applied in the linear analysis of piezoelectric beam and non-linear analysis of an aluminum cantilever beam attached with piezoceramics. Roccella and Gaudenzi [20] used quadratic variation of electric potential through the thickness suggested by [19] in the formulation of piezoelectric plate model. Gaudenzi [21] developed a higher order beam model and also discussed the edge effect at the free boundary of the adaptive structure.

In this paper, a higher order shear and normal deformation theory (HOST8) is developed for the analytical solution of piezoelectric plates under plane strain condition. Eight degrees of freedom are used to expand primary displacement field whereas exact variation of electric field is obtained in the piezoelectric layers by solving the governing second order ordinary differential equation satisfying electrical boundary conditions in the thickness direction. Results are compared with exact solutions [4]. Kant [22], Manjunatha and Kant [23,24], Kant and Swaminathan [25], have contributed extensively to the development of higher order shear and normal deformation theories. Recently, Kant et al. [26] developed a novel semi-analytical methodology using mixed variables, by maintaining the fundamental elasticity relations between stress, strain and displacements. The method satisfies the requirements of the through thickness continuity of transverse stresses and displacements. Results are presented for laminates simply supported on all edges and under cylindrical bending. Kant et al. [27–29] also presented a new partial discretization mixed FE formulation for general laminates with any boundary conditions. Present results for non-piezoelectric plates under plane strain condition are compared with the elasticity [35], semi-analytical [26,30], partial FE [27–30] and Reissner and Mindlin [10,11] (FOST) solutions.

2. Coupled plane strain formulation

2.1. Displacement model

A rectangular smart plate structure is shown in Fig. 1. The length of the plate is denoted by a along x direction and y side is infinite. The geometrical configuration of the plate is such that the thickness dimension is along z direction. The top and bottom layers of the plate are of piezoelectric materials, which act as distributed actuator and sensor. The core of the plate called, substrate, is purely elastic and has any number of elastic layers.

Displacement components $u(x,z)$ and $w(x,z)$ at any point in the plate are expanded in a Taylor's series to approximate the two- dimensional (2D) elasticity problem as a one-dimensional (1D) plate problem in cylindrical bending. The assumed displacement fields are as follows:

Model: HOST8

$$\begin{aligned} u(x,z) &= u_0(x) + z\theta_x(x) + z^2 u_0^*(x) + z^3\theta_x^*(x) \\ w(x,z) &= w_0(x) + z\theta_z(x) + z^2 w_0^*(x) + z^3\theta_z^*(x) \end{aligned} \tag{1}$$

Model: FOST

$$\begin{aligned} u(x,z) &= u_0(x) + z\theta_x(x) \\ w(x,z) &= w_0(x) \end{aligned} \tag{2}$$

The following strain vector $[\varepsilon]$ is given by strain displacement relationship as per classical theory of elasticity:

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \\ \gamma_{xy} \\ \gamma_{yz} \\ \gamma_{xz} \end{Bmatrix} = \left\{ \frac{\partial u}{\partial x} \quad \frac{\partial v}{\partial y} \quad \frac{\partial w}{\partial z} \quad \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \quad \frac{\partial v}{\partial z} + \frac{\partial w}{\partial y} \quad \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} \right\}^t \tag{3}$$

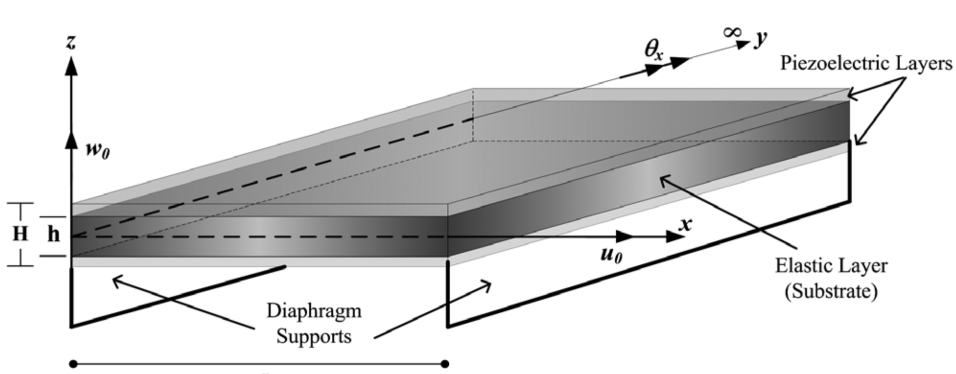

Fig. 1. Geometry of piezoelectric plate simply (diaphragm) supported on two infinite opposite edges and positive set of displacement components.