G Model
CULHER-3066; No. of Pages 10
ARTICLE IN PRESS

Journal of Cultural Heritage xxx (2016) xxx–xxx

Available online at
ScienceDirect
www.sciencedirect.com

Elsevier Masson France
EM|consulte
www.em-consulte.com/en

Numerical modelling of wooden structures

Daniel Konopka, Clemens Gebhardt, Michael Kaliske*

Technische Universität Dresden, Institute for Structural Analysis, D-01062 Dresden, Germany

ARTICLE INFO

Article history:
Received 2 March 2015
Accepted 16 September 2015
Available online xxx

Keywords:
Wood mechanics
Material modelling
Numerical simulation
Heat transport
Moisture transport
Inhomogeneity

ABSTRACT

Wooden structures are quite complex with respect to their material properties as well as to their structural response and, therefore, need to be simulated appropriately by means of numerical methods. This review provides an overview of current simulation techniques in static and dynamic analysis with respect to wood material models and their numerical realisation in their comprehensive complexity. The basic orthotropic elastic formulation of wood, a possible extension to a viscoelastic, viscoplastic formulation and the consideration of brittle failure are presented in terms of the finite element method, which is proposed as the preferred tool for the analysis of complex structures with highly nonlinear behaviour. Furthermore, models describing the dependency on climate conditions, long-term treatment and ageing are introduced. Since there is still a lack of understanding and a lack of data, it is adverted to further research effort in these domains. In the wide field of dynamic analysis of wooden structures, examples and approaches are presented. Subsequently, theories for taking into account the uncertain nature of wood in its micro- and macro-structure and a numerical example will round this review off.

1. Introduction

"The making of violins, cellos, pianos, and other musical instruments was an art long before being an object of scientific investigation. Architectural wood structures are artists' representations that rely on the advanced achievement of mechanics. The scientific knowledge of wood properties and characteristics is a necessary step toward its best use in artistic representations." With this quotation of Adriano Alippi [13], this review in numerical wood mechanics shall be introduced, since musical instruments show the entirety of the presented load and climate dependent mechanisms in wooden structures as an important field of application.

Complex structures, like music instruments, are the useful field of application for numerical analyses. A complex geometry and nonlinear material behaviour make an analytical solution impossible. Thus, numerical simulations by the finite element method (FEM) are needed. It is distinguished between static and dynamic investigations. Since the former is the special case of the latter one, it can be solved with less effort than the general dynamic analysis. The properties of wood remain the same, although more characteristics, like e.g. inertia and damping forces are needed for the latter case. For an appropriate simulation, eligible material models have to be applied to describe the material's behaviour and, thus, the structure's response to mechanical or climate load, static or dynamic load.

In this review, the most relevant material characteristics with focus on engineering parameters and their modelling in terms of an FE-analysis are presented. Due to the complexity of this issue, no guarantee for completeness is given. The models are just briefly introduced, but the references enable further studies.

2. Material modelling

Wood is an inhomogeneous, anisotropic and porous material with moisture-, temperature- and time-dependent behaviour [70]. In the scope of a realistic three-dimensional analysis by the FEM, appropriate material models for the description of the macroscopical, mechanical response to multi-physical loadings and their effects on all length scales from micro- to macro-scale are required. The material formulations are determined by deterministic engineering parameters like Young's moduli or material strengths as input quantities. They yield an appropriate deterministic solution for a structural analysis with the opportunity to apply uncertain as well as climate- and time-dependent input parameters. A comprehensive survey of experimental data can be found in [83].

2.1. Elasticity

A commonly used material model for the description of the approximately linear (in the range of small strains) and cylindrical

* Corresponding author. Tel.: +49 351 463 34386; fax: +49 351 463 37086.
E-mail addresses: daniel.konopka@tu-dresden.de (D. Konopka), clemens.gebhardt@tu-dresden.de (C. Gebhardt), michael.kaliske@tu-dresden.de (M. Kaliske).

http://dx.doi.org/10.1016/j.culher.2015.09.008
1296-2074/

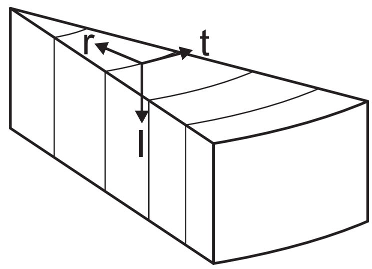

Fig. 1. Cylindrically anisotropic material directions of wood.

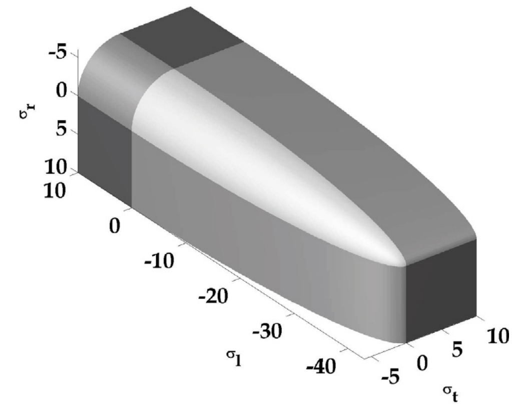

Fig. 2. Yield surface of the multi-surface plasticity model for compression strengths $f_{cr} = f_{ct} = -6\,\mathrm{N/mm^2}$, $f_{cl} = -43\,\mathrm{N/mm^2}$ and $q = 0$ and $\sigma_r = \sigma_t = [-6\,;\,10]\,\mathrm{N/mm^2}$ and $\sigma_l = [-43\,;\,10]\,\mathrm{N/mm^2}$ (see [87]).

anisotropic behaviour is orthotropic elasticity (e.g. [86,88,92]). The linear relation between elastic strain $\underline{\boldsymbol{\varepsilon}}$ and resulting stress $\underline{\boldsymbol{\sigma}}$ is given by Hooke's law. For the three-dimensional case, it is denoted as

$$\underline{\boldsymbol{\sigma}} = \underline{\underline{\mathbf{C}}} : \underline{\boldsymbol{\varepsilon}}. \tag{1}$$

With the material directions radial, tangential and longitudinal [r, t, l] (see Fig. 1) and assuming a symmetric, elastic material tensor $\underline{\underline{\mathbf{C}}}$, it can be defined using Young's moduli E_i, shear moduli G_{ij} and Poisson's ratios ν_{ij}, $i, j \in \{r, t, l\}$

$$(\underline{\underline{\mathbf{C}}})^{-1} = \begin{bmatrix} \frac{1}{E_r} & -\frac{\nu_{rt}}{E_t} & -\frac{\nu_{rl}}{E_l} & 0 & 0 & 0 \\ -\frac{\nu_{rt}}{E_t} & \frac{1}{E_t} & -\frac{\nu_{tl}}{E_l} & 0 & 0 & 0 \\ -\frac{\nu_{rl}}{E_l} & -\frac{\nu_{tl}}{E_l} & \frac{1}{E_l} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{G_{rt}} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{G_{tl}} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{G_{rl}} \end{bmatrix}. \tag{2}$$

Although the elastic behaviour is more complex and yields a nonsymmetric compliance, as can be seen by experiments (e.g. [68]), Eq. (2) is an appropriate and useful model [10].

2.2. Ductile failure

Strength properties of wood significantly differ for compressive, tensile and shear loading. After exceeding the compression strength, that often is defined as the end of the elastic range in material modelling (e.g. [83,86,92]), wood shows a distinct plastic behaviour. A comprehensive overview of uniaxially loaded wood is given in [53,70]. Extensive studies on strength properties of biaxially loaded spruce wood are published in [19,23]. The theoretical basics of inelastic behaviour and the numerical implementation can be found in [99]. First applications of plasticity models in timber material mechanics are described in e.g. [63]. Although wooden constructions are built to bear loads in a uniaxial manner parallel to the grain, complex structures are usually loaded multiaxially by constraints or multiaxial spatial external forces. Thus, one of the most recent models in wood mechanics deals with a three-dimensional multi-surface plasticity formulation. Since in [23,63], a biaxial formulation is presented, a three-dimensional multi-surface plasticity formulation is developed in [87], based on previous approaches [92,95]. With regard to the Tsai-Wu failure criterion [109], the yield criterion identifies the elastic and ductile zones by

$$f(\underline{\boldsymbol{\sigma}}, q) = \underline{\boldsymbol{\sigma}} : \underline{\underline{\mathbf{b}}} : \underline{\boldsymbol{\sigma}} + q(\alpha) - 1 \leq 0. \tag{3}$$

The admissable stress state, i.e. elastic zone, is found for negative values of $f(\underline{\boldsymbol{\sigma}}, q)$. Inelastic deformations are evoked if the stresses exceed the elastic limits, thus, values are larger than zero. The formulation fulfils a C_1-continuous transition between the single yield surfaces, because it does not contain a linear term and, thus, enables a proper application of the Newton–Raphson algorithm to find the equilibrium path. The ductile behaviour of wood cannot be characterised as ideal plastic. Thus, the post yield portion of the stress–strain curve has to be identified with softening or hardening features. In [87], hardening after ductile failure is included with the help of the scalar term $q(\alpha)$ representing the hardening potential, a function of the inner strain-type variable α which describes the current hardening state. The tensor containing the strengths of the material is defined by

$$\underline{\underline{\mathbf{b}}} = \underline{\mathbf{b}}_s \otimes \underline{\mathbf{b}}_f, \tag{4}$$

with the tensor $\underline{\mathbf{b}}_s$ allocating the stress octant, for which the yield surface has to be defined, depending on the current stress state. The tensor $\underline{\mathbf{b}}_f$ contains the compression strengths, i.e. the elastic limits, for the material directions

$$\underline{\mathbf{b}}_f = \begin{bmatrix} \frac{1}{f_{cr}^2} & 0 & 0 \\ 0 & \frac{1}{f_{ct}^2} & 0 \\ 0 & 0 & \frac{1}{f_{cl}^2} \end{bmatrix}. \tag{5}$$

The plasticity formulation has to fulfil the Kuhn–Tucker complementary conditions and is described by an associated flow rule in the present case. The overall strain (see Eq. (1)) is the sum of both, elastic and plastic strain

$$\underline{\boldsymbol{\varepsilon}} = \underline{\boldsymbol{\varepsilon}}^e + \underline{\boldsymbol{\varepsilon}}^p. \tag{6}$$

Fig. 2 shows the assembled yield surfaces ($q = 0$), whereas the light grey area describes the stress octant for the combination of all three failure modes. The middle grey areas describe two failure modes and the dark grey areas correspond to a single failure mode, respectively.

2.3. Brittle failure

In the previous chapter, ductile mechanical behaviour of wood is introduced. Due to the micro-structure of wood formed by cells and fibres, wood shows brittle failure behaviour for tensile and shear stresses. In both situations, the cells will suddenly collapse at high