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Construction of micropolar continua from the asymptotic homogenization of beam lattices

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ABSTRACT

The asymptotic homogenization of periodic beam lattices is performed in an algorithmic format in the present contribution, leading to a micropolar equivalent continuum. This study is restricted to lattices endowed with a central symmetry, for which there is no coupling between stress and curvature. From the proposed algorithms, a versatile simulation code has been developed, relying on an input file giving the lattice topology and beam properties, and providing as an output the equivalent stiffness matrix of the effective continuum. The homogenized moduli are found in close agreement with the moduli obtained from finite element simulations performed over extended lattices. The obtained results are exploited to design and calculate a lattice endowed with a hierarchical double scale microstructure, leading to a dominant micropolar effect under bending at the macroscopic scale.

1. Introduction

Lattices endowed with a specific mechanical behavior due to the presence of an inherent microstructure continue to attract the interest of researchers [1]. The relationship between the material microstructure and the resulting properties is the key to optimization and design of lightweight, strong, and tough materials and structures [2].

An important category consists of lattices having a discrete kinematics and topology, leading to a micropolar effective behavior at the macroscopic scale of description. Such effects have been proven to be important when specimen dimensions are comparable with the cell size [3]. However, these effects are not easy to evidence from a direct analysis at the macroscopic scale. The first motivation of such micromechanically inspired analysis is an increased understanding of the behavior of those lattices in certain loading situations (concentrated forces, tolerance to damage, perforations) [4], certain geometries [5,6], or when submitted to heat exchanges [7,8]. For example, contributions [9,10] show that the variation of the stress concentration at the interface between bone and prosthesis can be explained by the micropolar structure of the medium. A second motivation of homogenization techniques is their use as a tool to conceive and analyze novel structural materials exhibiting unconventional mechanical properties or behaviors [11,12]. A third motivation of deriving macroscopic models of lattices at an intermediate scale is the reduction of the induced computational cost.

The homogenization of lattices towards a micropolar continuum has been an active research field since a long time [13], and several methods have been developed for that purpose [14]: the finite difference method [15], energy equivalence concepts, the potential and kinetic energies of a typical cell of the lattice are equated to those of the continuum, after expanding the nodal displacements of the lattice in a Taylor series [16], averaging methods [17,18], solving fundamental boundary-value problem using symmetry of the repetitive microstructure [19,20], exploring the response for various boundary conditions [21], using Korn-type inequality [22].

One method that has been prolific is the double-scale asymptotic expansion; its principle goes back to the work of Sanchez–Palencia [23]. This approach was applied to reticulated structures by Cioranescu and Saint-Jean Paulin [24] and to other periodical structures by Boutin [25]. Several variations were then developed for the homogenization of beam lattices. Pradel and Sab [26,27] suggested to treat homogenization of beam lattice in a way similar to the homogenization of media made of discrete particles; the resolution of the unknowns is done by those authors by minimizing an energy functional. In the work of Caillerie et al. [28,29], the authors develop a method which uses only one variable in the asymptotic expansions. The work of those authors is however limited to lattices endowed mainly with an extensional behavior. Boutin et al. [30,31] propose a dynamical formulation of the balance of forces to solve vibration problems, applied essentially to square unit cells.

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Nomenclature

$\mathbb{B}$	set of beams of an elementary cell
δ^i	shift factor for nodes belonging to a neighboring cell
E_s	elastic modulus of structural material
E_h	homogenized elastic modulus
$\mathbf{e}_i^\lambda$	unit vectors of the curvilinear coordinate system associated to the lattice (generally non cartesian)
$\mathbf{e}_i$	vector of the cartesian coordinate system
$\varepsilon = l/L$	small scale parameter
$\boldsymbol{\epsilon}$	deformation tensor
γ	micropolar modulus
g	Jacobian of the transformation from dx to $d\lambda$
H	width of a microstructured beam
I^b	second moment modulus of beam b
I_h	second moment modulus of a microstructured beam
$[k]$	matrix of the curvature of a beam in local coordinates
$[K]$	stiffness matrix of the homogenized medium
$[K_{fl}]$	stiffness matrix of the bending of a beam in local coordinates
$[K_{tc}]$	stiffness matrix of the extension of a beam in local coordinates
$[K_b]$	complete stiffness matrix (sum of $[K_{fl}]$ and $[K_{tc}]$) of a beam in local coordinates
K_f^m	bending stiffness of a microstructured beam
K_f^s	bending stiffness of a homogeneous standard beam
$\boldsymbol{\kappa}$	curvature tensor
κ	micropolar modulus
k_l	stiffness of a beam in extension
k_f	bending stiffness of a beam
L	characteristic length of the whole structure
l	characteristic length of the elementary cell
$l^b = \varepsilon L^b$	length of the beam b
λ^i	curvilinear coordinates associated with the vectors $\mathbf{e}_i^\lambda$
$\mathbf{m}$	couple stress tensor
$\mathbf{M}^n$	moment at node n
μ	classical Lamé shear modulus
$\boldsymbol{\mu}^i$	couple stress vectors
μ^m	micropolar shear modulus
$\mathbf{N}^b = N^b\mathbf{e}^b$	normal forces of the beam b
$\mathbb{N}$	set of nodes of an elementary cell
ν_h	homogenized Poisson's ratio
ϕ^n	angular displacement at node n
$\tilde{\phi} = \frac{1}{2}(\phi^{E\varepsilon} + \phi^{O\varepsilon})$	mean angular rotation associated to the beam's node
$\hat{\phi} = (\phi^{E\varepsilon} - \phi^{O\varepsilon})$	angular difference associated to beam's node
$\boldsymbol{\phi}_0$	homogenized angular displacement of the equivalent medium at zeroth order
$\boldsymbol{\sigma}$	Cauchy stress tensor
t^b	width of the beam b
O, E	the origin and end node of a beam respectively
P^*	virtual power for one beam
$\mathbf{R}(\lambda^1,\lambda^2)$	continuum node position
$\mathbf{S}^i$	stress vectors
$[S]$	compliance matrix
$\mathbf{T}^b = \mathbf{T}^E = -\mathbf{T}^O$	resultant of forces at the nodes of a beam b
$\mathbf{T}_t^b = T_t^b\mathbf{e}^{b\perp}$	shear forces of the beam b
$\mathbf{U}^n = (u^n, v^n)$	displacement vector at node n
$\mathbf{V}^{n*}$	virtual translational velocity field
$\tilde{v} = \frac{1}{2}(v^{E\varepsilon} + v^{O\varepsilon})$	mean homogenized bending components associated to the beam's node
$\hat{v} = (v^{E\varepsilon} - v^{O\varepsilon})$	difference of homogenized bending components associated to the beam's node
v_l	beam's local bending function, satisfying Bernoulli's assumptions
W	length of a macroscopic beam
W_i^*	virtual work of internal forces
$\boldsymbol{\omega}^{n*} = \omega^{n*}\mathbf{e}_3$	virtual rotational velocity field at node n
x^i	coordinates associated with vectors $\mathbf{e}_i$
$\mathbb{Z}$	set of structural cells
Notations	We have adopted specific notations exposed in the sequel. Several quantities are sometimes indexed by the superscript $^\varepsilon$, which refers to the asymptotic expansion of a variable (the same variable may also appear without this superscript). For example, the displacement $\mathbf{U}$ may appear either as $\mathbf{U}^n$, the standard displacement at node n, or as the following expansion versus $\varepsilon, \mathbf{U}^{n\varepsilon} = \mathbf{U}_0 + \varepsilon\mathbf{U}_1^n + \varepsilon^2\mathbf{U}_2^n + \ldots$ The lowest order term is uniform within the unit cell, and is therefore not dependent on the specific node n. In order to distinguish the virtual quantities from other quantities, we will add the superscript * for the former.

All the previously mentioned asymptotic methods have similarities. In a previous paper [32], we have shown the interest of the variant introduced by Caillerie et al. [33] for the homogenization of beam lattices with negative Poisson's ratios towards a classical continuum. In the present contribution, we extend this approach for the automatic homogenization of beam lattices having a complex unit cell towards a micropolar effective medium. We shall restrict the forthcoming developments to quasi static 2D situations, adopting a small deformations framework in the elastic range. An essential objective of this work is to develop a homogenization procedure in an algorithmic format for general beam lattices, leading to an effective micropolar behavior.

The paper is organized as follows: the discrete homogenization technique will be exposed in Section 2. In Section 3, the developed method is validated and an algorithm is proposed and applied to the square and hexagonal lattices. Thereafter, as an illustration of the powerfulness of the homogenization technique, a beam lattice having a two-scales microstructure is conceived, exhibiting a micropolar dominant effect at the macroscopic scale under bending. A summary of the main achievements and perspectives is given in Section 4.

2. Homogenization of lattices and construction of effective micropolar continua

In this section, we briefly recall the constitutive equations of micropolar continua and discuss the choices made for the asymptotic developments of the kinematic variables.

2.1. The micropolar theory

In the theory of classical continua, the displacement is the sole degree of freedom u_i representative of translations, and the induced small strain measure is then $\epsilon_{ij} = \frac{1}{2}(u_{j,i} + u_{i,j})$. The idea of couple stress was explored in the middle of the 19th century by (MacCullagh (1839), Lord Kelvin (1882–1890), Voigt (1887)), and was pursued later on by the Cosserat brothers [34], who proposed a theory based on a rigid triad of vectors attached to each point of a continuum, endowed with a local rotation considered as independent from the local rotation due to the deformation. In the so-called micropolar (or Cosserat) theory, two sets of degrees of freedom are present, namely the displacement and the rotation fields, defining the set of d.o.f. $(\mathbf{u},\boldsymbol{\phi})$. The pioneering work of the