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Geometrically exact assumed stress–strain multilayered solid-shell elements based on the 3D analytical integration

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Abstract

This paper presents a family of geometrically exact assumed stress–strain four-node curved solid-shell elements with six displacement degrees of freedom per node by using the first-order equivalent single-layer theory. The proposed finite element formulation is based on the new strain–displacement relationships written in general reference surface coordinates, which are objective, i.e., invariant under rigid-body motions. This is possible because displacement vectors of the bottom and top surfaces of the shell are introduced and resolved in the reference surface frame. To overcome shear and membrane locking and have no spurious zero energy modes, the assumed strain and stress resultant fields are invoked. In order to circumvent thickness locking, three types of the modified material stiffness matrix extracted from the literature are employed and compared. All three elemental stiffness matrices have six zero eigenvalues and require only direct substitutions. Besides, they are evaluated by applying the 3D analytical integration that is very economical and allows using extremely coarse meshes.

Keywords: First-order shell theory; Assumed stress–strain element; Analytical integration

1. Introduction

In the last 15 years, a considerable work has been carried out on three-dimensional continuum-based finite elements that can handle shell analysis satisfactorily. These elements are typically defined by two layers of nodes at the bottom and top surfaces of the shell with three displacement degrees of freedom per node and known as *isoparametric* solid-shell elements [1–10]. In the isoparametric solid-shell element formulation, initial and deformed geometry are equally interpolated allowing one to describe rigid-body motions precisely. The development of solid-shell elements is not straightforward. In order to overcome element deficiencies such as shear, membrane, trapezoidal and thickness locking, advanced finite element techniques including assumed natural strain, assumed strain, enhanced assumed strain and hybrid stress methods have to be applied. Still, the isoparametric solid-shell element formulation is computationally inefficient because stresses and strains are analyzed in the global or local orthogonal Cartesian coordinate system, although the normalized element coordinates represent already curvilinear convective coordinates.

An alternative way is to develop the *geometrically exact* solid-shell element based on the *general* curvilinear coordinates that finds its point of departure in papers [11–15], in which only orthogonal curvilinear reference surface coordinates were employed. The term "geometrically exact" reflects the fact that reference surface geometry is described by analytically given functions. Such elements are very promising due to the fact that in the geometric modeling of modern CAD systems the surfaces are usually generated by non-uniform rational B-splines (NURBS) [16]. Allowing for that surfaces are conventionally produced by the position vector with representation of two parameters, we can connect the geometric modeling of the shell surface generated in the CAD system to the finite element analysis

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of shell structures. So, it is advantageous to use NURBS shell surface functions directly in the shell calculations and geometrically exact solid-shell elements are best suited for this purpose. They also have the two-parameter representation in surfaces and all geometric computations may be done in the reference surface using NURBS surface representations in the CAD system.

The solid-shell element formulation developed is based on the principally new strain–displacement relationships of the first-order equivalent single-layer (ESL) theory, written in the general reference surface coordinates. It is remarkable that these strain–displacement relationships *precisely* represent all rigid-body motions and no assumptions except for standard Timoshenko–Mindlin kinematics are required to derive them. For this purpose displacement vectors of the bottom and top surfaces of the shell are introduced but resolved, in contrast with the isoparametric solid-shell formulation [1–10], in the reference surface frame. One can compare the proposed shell formulation with a close formulation of Simo et al. [17], where geometrically exact solid-shell elements have been also developed but displacement vectors are resolved in the global Cartesian basis and, therefore, Christoffel symbols and coefficients of the second fundamental form do not explicitly appear in the formulation. This restriction does not give an opportunity to employ the above NURBS surface function technique directly.

The finite element formulation is based on the simple and efficient approximation of shells via four-node curved shell elements. To avoid shear and membrane locking and have no spurious zero energy modes, the assumed stress resultant and displacement-independent strain fields are invoked. This approach was proposed by Wempner et al. [18] for the classic first-order shear deformation shell theory and further was generalized by Kulikov and Plotnikova [14,15] for the so-called Timoshenko–Mindlin shell theory allowing for the thickness change.

It is known that a six-parameter shell formulation on the basis of the complete 3D constitutive equations is deficient because thickness locking can occur [10]. To prevent thickness locking at the *finite element level* an efficient enhanced assumed strain method [3,4] can be applied. In order to circumvent a locking phenomenon at the *mechanical level* and computational one as well, the 3D constitutive equations have to be modified. For this purpose three simple and effective remedies may be employed, namely, the *ad-hoc* modified laminate stiffness matrix [9,19,20] and simplified material stiffness matrices symmetric [1,2,6,14,21] or nonsymmetric [15,22,23] corresponding to the generalized plane stress condition. Herein, all remedies are introduced into the formulation that allows one to assess their advantages and disadvantages.

Taking into account that displacement vectors of bottom and top surfaces of the shell are represented in the reference surface frame, the proposed geometrically exact solid-shell formulations have computational advantages compared to the conventional isoparametric finite element formulations, since they reduce the costly numerical integration by deriving the stiffness matrices. Besides, element matrices developed require only direct substitutions, i.e., no inversion is needed if sides of the element coincide with lines of principal curvatures of the reference surface and they are evaluated by using the 3D analytical integration. As it turned out, an analytical integration leading to the elemental stiffness matrix is very economical and performs well even for the extremely coarse meshes.

2. Geometry and kinematic description of shell

Consider a shell built up in the general case by the arbitrary superposition across the wall thickness of N layers of uniform thickness h_k. The kth layer may be defined as a 3D body of volume V_k bounded by two surfaces S_{k-1} and S_k, located at the distances δ_{k-1} and δ_k measured with respect to the reference surface S, and the edge boundary surface Ω_k (Fig. 1). The full edge boundary surface $\Omega = \Omega_1 + \Omega_2 + \cdots + \Omega_N$ is generated by the normals to the reference surface along the bounding curve $\Gamma \subset S$. It is also assumed that the bounding surfaces S_{k-1} and S_k are continuous, sufficiently smooth and without any singularities. Let the reference surface S be referred to the general curvilinear coordinate system α^1 and α^2. The coordinate α^3 is oriented along the unit vector $\boldsymbol{a}_3$ normal to the reference surface, while $\boldsymbol{a}_1$ and $\boldsymbol{a}_2$ denote the covariant basis vectors of the reference surface.

The constituent layers of the shell are supposed to be rigidly joined, so that no slip on contact surfaces and no separation of layers can occur. The material of each con-

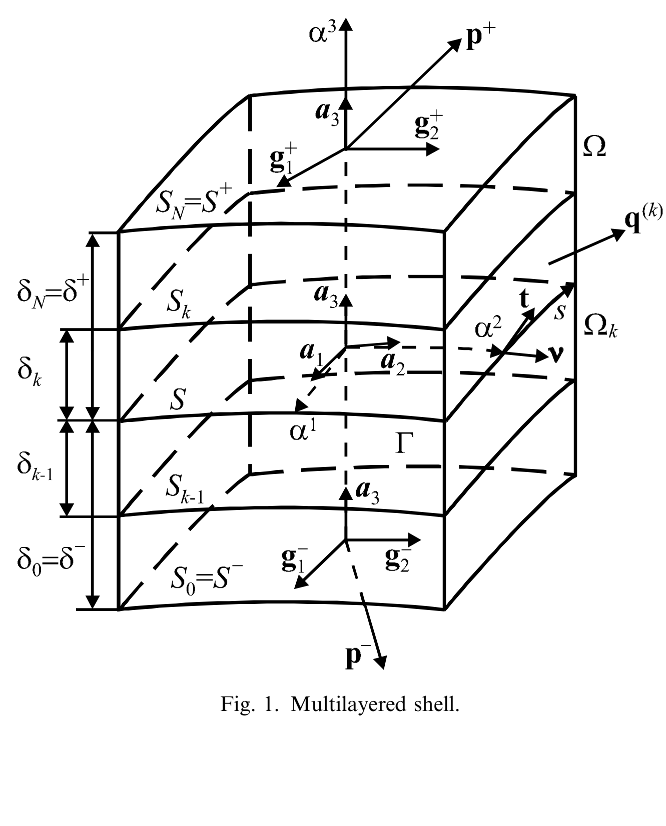

Fig. 1. Multilayered shell.