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Calculation of surface energy associated with formation of multiple kinked cracks

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Abstract

An energy parameter termed M_c is proposed for calculation of the surface energy associated with formation of multiple kinked cracks in 2-D anisotropic elastic solids. The formulation is based on a path-independent contour integral. The surface energy can be exactly evaluated by M_c when the cracked solids are homogeneously-stressed (i.e., the material characteristic), and effectively approximated when the solids are nonhomogeneously-stressed (i.e., the structural characteristic). It is thus suggested that M_c be possibly used as a fracture parameter that quantitatively measures the degradation of material (e.g. in micromechanics) and/or structural integrity due to irreversible evolution of multiple kinked cracks.

Keywords: Multiple kinked cracks; Surface energy; M_c-integral; Path-independence; Material characteristic; Structural characteristic

1. Introduction

The strength of many engineering components is significantly limited by the growth of a system of distributed cracks rather than a single, continuous crack. Such multi-defect fracture conditions range from the presence of a finite number of macrocracks, to the formation of densely distributed microcracks with random location and orientation. Furthermore, the cracks may have irregular kinked shapes, each consisting of various number of segments with different lengths and orientations. Typically, two major issues are concerned in the case of multiple kinked cracks. One is investigation of the fracture criteria associated with local conditions that drive growth, kinking, interaction, and coalescence of the cracks (e.g. [1–7], etc.). The other is examination on loss of global mechanical integrity caused by evolution of the multiple cracks in a medium. Although studies regarding the second issue have been extensively reported in the literature, most of them are considered with the presence of multiple straight cracks. For example, the interactions among periodically distributed parallel straight cracks in a cracked body were evaluated asymptotically in terms of the effective elastic moduli by Wang et al. [8,9]. In addition, review works on other related studies in the context of micromechanics have been presented by Kachanov [10], Petrova et al. [11], etc. As to problems containing multiple kinked cracks, more investigations on the associated global fracture behavior are still in need.

The use of energy parameters in characterizing fractural behavior is of practical interest, especially when the stress states in the region around the singular points are too complicated to be analytically described. For problems containing a single crack, the J_1-integral has been widely used as an energy fracture parameter that evaluates the energy release rate associated with extension of the crack tip in quasi-brittle materials. In addition to J_1, the M-integral (another energy conservation contour integral derived from Noether's theorem) has also found considerable applications (e.g. [12–14]), Conventionally, by taking a counterclockwise contour enclosing a straight crack in a

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homogeneously-stressed media, integration of M leads to the energy release rate due to self-similar expansion of the crack. By comparing with J_1, the M-integral appears to be more ‘global’ in nature since its application corresponds with the whole crack rather than a single tip. Such a global feature implies the applicability of M for use in multi-cracked problems.

The M-integral has recently been applied to problems containing multiple straight cracks. A series of studies were presented by using M to characterize the material damage level due to microcracking in homogeneously-stressed infinite elastic media [15–17]. Also, a problem-invariant parameter M_c is proposed by the authors [18,19] to describe the degradation of material and/or structural integrity caused by irreversible evolution of multiple cracks. An important contribution in these works is addressed to illustration of the physical meaning of M_c, which evaluates twice the surface energy associated with formation of all the cracks. With such interpretation, the M_c-integral is thus suggested to be used as an energy parameter that quantitatively measures the fracture level corresponding to stiffness loss in the multi-cracked body.

In this present paper, an energy parameter termed M_c is developed for evaluation of the surface energy associated with formation of multiple kinked cracks in 2-D elastic solids. This is an extension of the aforementioned earlier work conducted by the authors on the corresponding straight cracked problems. The M_c-integral is formulated by modifying the concept of the M-integral, in which the integration contour and the reference coordinate system are suitably selected. The integration appears to be path-independent so that the complicated near-tip and near-kink singular behavior need not be considered in its calculation. The proposed formulation is feasible for problems modeled with generally anisotropic materials. Although most of the related research works reported in the literature are concerned with degradation of material integrity due to presence of multiple cracks, applicability of M_c with regard to both material and structural characteristics will be addressed in this present work.

2. The M_c-integral

In this section, the basic concept of the contour integral M_c is introduced. We first consider the instance of a single kinked crack, and then extend to the condition containing multiple kinked cracks.

2.1. A single kinked crack

We consider a 2-D anisotropic elastic body, containing a single kinked crack with two tips P and Q. The crack consists of m segments, each of length l^i $(i = 1,\ldots,m)$ and with different orientations, as shown in Fig. 1 ($m = 4$ in this figure). By introducing a coordinate system originating at an arbitrarily chosen point O and, with no loss of generality, the first segment of crack lying parallel to the x_1-direction, When the body is subjected to external loads and no body forces, the M-integral is defined as

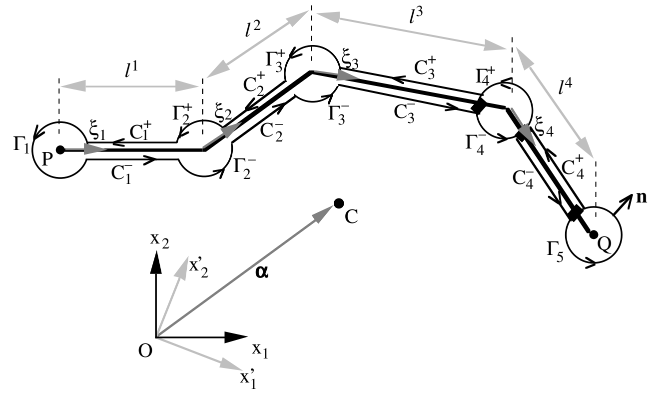

Fig. 1. A kinked crack located in an anisotropic elastic body. The crack consists of m segments ($m = 4$ in this figure).

$$M = \int_D \left[W n_k x_k - T_i \left(\frac{\partial u_i}{\partial x_k} \right) x_k \right] \mathrm{d}s, \tag{1}$$

where W is the strain energy density of the material, $\mathbf{T}$ is the traction vector, $\mathbf{u}$ is the displacement vector, $\mathbf{n}$ is the outward unit vector normal to the integration contour D, $\mathbf{x}$ is the position vector of the integration point, and s is the arc length along D. Due to the presence of singularities at the crack tips and the kinked points (as addressed in [1–7]), D is thus defined as a counterclockwise closed contour around the whole crack, which consists of $4m$ curve segments as $\Gamma_1 + C_1^- + [\sum_{i=2}^{m} (\Gamma_i^- + C_1^-)] + \Gamma_{m+1} + [\sum_{i=0}^{m-2} (C_{m-i}^+ + \Gamma_{m-i}^+)] + C_1^+$. Note that, by definition, the integration is carried out with the limiting condition where $\Gamma_1, \Gamma_{m+1}, \Gamma_i^-, \Gamma_i^+$ $(i = 2,\ldots,m)$ are shrunk onto the two crack tips (i.e., P and Q) and the kinked points respectively. Also, C_i^- and C_i^+ $(i = 1,\ldots,m)$ are lying along the crack surfaces (this limiting condition is not shown in Fig. 1).

When the body is subjected to a nonhomogeneous stress field, it can be shown (Appendix I) that the value of M varies with the selection of origin O. Nevertheless, by locating the origin at the geometric center C of the two crack tips, i.e., by taking $\boldsymbol{\alpha} = (0, 0)$, we can then define a problem-invariant parameter M_c as

$$M_c = M|_{\boldsymbol{\alpha}=(0,0)}. \tag{2}$$

Note that the kinked points are not included in positioning the geometric center C in that they make no contribution to the integration of M (Appendix I).

2.2. Multiple kinked cracks

Consider the 2-D anisotropic body containing N distributed kinked cracks, each consisting of m_r $(r = 1,\ldots,N)$ segments (of length l_r^i $(i = 1,\ldots,m_r)$), with random location and orientation, as shown in Fig. 2 ($N = 5$ in this figure). The geometric center of all the crack tips (i.e., P_r and Q_r, $r = 1,\ldots,N$) is positioned and denoted C.