Available online at www.sciencedirect.com
SCIENCE DIRECT®
ELSEVIER
Computers and Structures 83 (2005) 1902–1916
Computers & Structures
www.elsevier.com/locate/compstruc

Evolutionary structural optimisation based on boundary representation of NURBS. Part I: 2D algorithms

E. Cervera, J. Trevelyan *

School of Engineering, University of Durham, South Road, Durham DH1 3LE, UK

Received 21 June 2004; accepted 16 February 2005
Available online 19 April 2005

Abstract

This paper presents an evolutionary structural optimisation (ESO) approach based on the boundary element method. Non-uniform rational B-splines (NURBS) are used to define the geometry of the component and, since the shape of these B-splines is governed by a set of control points, use can be made of the locations of these control points as design variables. The developed algorithm creates NURBS-based internal cavities to accomplish topology changes. The optimum topologies evolve allowing cavities to merge between each other and to their closest outer boundary. Two-dimensional structural optimisation is investigated in detail exploring single and multiple load case elastic problems.

Keywords: Evolutionary structural optimisation; Boundary element method; NURBS

1. Introduction

In shape optimisation, the topology of the structure is fixed and only the shape of the boundary can change. Extensive work has been done in shape optimisation [1] using numerical methods, such as the finite element method (FEM) [2,3] and the boundary element method (BEM) [4], for structural analysis. Of special interest to this work has been the application of BEM when design sensitivity analysis is employed in the optimisation algorithm [4]. This is due to the high accuracy of the BEA results on the boundary. Indeed, since the method deals with integrals over the boundary, only the boundary needs to be discretised which is a clear advantage for remeshing purposes. Heuristic methods have been also applied to shape optimisation. These methods are known as gradientless methods since they require no sensitivity calculations. Schnack and Spörl [5] introduced a method for the reduction of stress concentration on boundaries through the gradual removal of low stress material. In this fashion, the biological growth method [6] is based on the hypothesis that in nature structures such as trees have a constant stress distribution over the boundaries.

In the field of structural optimisation, topology optimisation refers to optimal design problems in which the topology of the structure is allowed to change in order to improve the performance of the structure. For that reason, this optimisation class is regarded as one of the most challenging optimisation problems. Much research has been devoted to topology optimisation over the last 10 years; this has included methods based on microstruc-

* Corresponding author. Tel.: +44 191 334 2522; fax: +44 191 334 2390.
E-mail address: jon.trevelyan@durham.ac.uk (J. Trevelyan).

0045-7949/$ - see front matter © 2005 Elsevier Ltd. All rights reserved.
doi:10.1016/j.compstruc.2005.02.016

tures such as the homogenisation method [7] and the solid isotropic microstructure with penalty (SIMP) method [8], and methods that deal with elements on a macro basis approach such as the soft-kill option (SKO) [9], the evolutionary structural optimisation (ESO) method [10] and the bubble method [11]. Hassani and Hinton [12] present a useful review of topology optimisation methods. More recently, methods such as genetic algorithms (GAs) [13] have been applied to topology optimisation since they are particularly robust in finding global optima. However, this can come at a high computational cost. Also, the concepts of robust and reliable design [14] are of growing interest in topology optimisation since they consider input variations and uncertainties in the design and manufacturing process.

From the different topology optimisation techniques presented, the evolutionary structural optimisation method ESO [10] is chosen as the basis for the current research. Most of the work carried out on ESO is based on the FEM. The classical ESO [15] approach is based on the idea of removing inefficient material from an initially oversized domain. The removal process is carried out by deleting regions occupied by elements with low stresses. By repeating this process and removing small amounts of material at each stage, the topology for the structure gradually evolves to a more efficient structure. Following this basic approach, there have since been a number of modifications and refinements such as BESO [16] where not only are elements removed but are also added in high stressed areas. Recent applications of ESO [17] cover a wide range of physical situations by considering element modification sensitivity terms. These sensitivities are used to drive the removal and addition process in order to achieve a minimum (or maximum) of the objective function.

Nevertheless, ESO has some drawbacks and weaknesses, notably those related to the rejection criteria, as has been reported by Zhou and Rozvany [18], those due to mesh dependency and those due to the tendency to converge to meshes exhibiting an undesirable single corner contact [19]. Moreover, the final solution, due to the nature of the mesh, can result in jagged edges. It is clear that this is an undesirable situation, since the accuracy of the FE results is in doubt. In addition, post-processing, such as spline construction, must be carried out to smooth the boundary for manufacturing reasons [20]. Similar ideas are presented by Maute and Ramm [21] and Hammer and Olhoff [22] in the material topology optimisation approach, in which the geometry is smoothed using splines based on density distributions. Another disadvantage, common in fixed grid FE-based structural optimisation methods, is the presence of checkerboard patterns [23]. These refer to the phenomenon of alternating presence of void and solid elements ordered in a checkerboard fashion. Such patterns result in structural analysis inaccuracies which complicate the interpretation of optimal material distribution and geometry extraction. Checkerboard formation is related to the use of low-order elements in the finite element approximations. The use of higher order elements can reduce this effect, but at the cost of increasing the computational time required. Alternatively, to improve the estimation quality of elemental sensitivity in low-order elements, a smoothing algorithm [23] can be implemented. On the other hand, in the ESO-based intelligent cavity creation (ICC) algorithm [24] checkerboard patterns (with numerous cavities) can be eliminated through controlling the number and scale of structural cavities in the final topology. Alternatively, a perimeter control technique [25] can be incorporated into BESO to overcome checkerboard patterns. This technique has also been shown to reduce the dependency of the converged optimum on the initial finite element discretisation.

Finally, an important task pointed out by Querin et al. [16] is related to the solution time, that is, the necessity to make the ESO process faster so that the designer is able to obtain the optimum design within a few seconds of describing the environment. Research in this topic has been carried out combining ESO and the fixed grid (FG) method [24], instead of classical FE, to improve the performance.

In order to overcome these drawbacks relating largely to domain-based discretisation, the boundary element method (BEM) [26] may be implemented as the tool to carry out structural analysis. In addition, in the current work the geometry is described using the non-uniform rational B-spline curves (NURBS) [27]. The optimisation process is fully integrated within in-house boundary element software [28] allowing a straightforward and rapid communication between the analysis software and optimisation code since they share the same database. The advantages of the proposed algorithm over FEM based ESO procedures are, therefore, the easy remeshing and smoothness of geometry that provides for a smoothness in the stress solution. The use of boundary elements enables further computational efficiency gains beyond a smoothed FE approach (e.g. [21,22]) since many computations may be saved by reusing influence coefficients relating to the boundaries that remain unchanged in successive iterations. The use of the BEM, however, restricts the class of problems to those to which are suitable for BEM treatment. The present work is limited to linear elastostatics, but this might be extended through, for example, dual reciprocity methods, to non-linear problems.

2. Algorithm

The developed algorithm considers the BEM to carry out the structural analysis of the component under study. The optimisation approach is stress-based selecting low