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Peak response of non-linear oscillators under stationary white noise

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Abstract

The use of the advanced censored closure (ACC) technique, recently proposed by the authors for predicting the peak response of linear structures vibrating under random processes, is extended to the case of non-linear oscillators driven by stationary white noise. The proposed approach requires the knowledge of mean upcrossing rate and spectral bandwidth of the response process, which in this paper are estimated through the stochastic averaging method. Numerical applications to oscillators with non-linear stiffness and damping are included, and the results are compared with those given by Monte Carlo simulation and by other approximate formulations available in the literature.

Keywords: Censored closure; Computational stochastic mechanics; First passage time; Gumbel distribution; Poisson approach; Random vibration; Reliability analysis; Stochastic averaging

1. Introduction

The stochastic analysis of structural and mechanical systems subjected to dynamic actions of a random nature has become very popular in the last decades, given that in a number of engineering situations deterministic approaches are quite unsatisfactory.

When the dynamic excitation is modelled as a Gaussian process, and the system exhibits a linear behaviour, the response is Gaussian too. In this case, then, the knowledge of mean value and standard deviation fully defines the response from a probabilistic point of view. In many circumstances, however, due to a non-linear behaviour of the system, the response may significantly deviate from the Gaussianity, and higher-order statistics are then required. Unfortunately, these are available in exact form just for a restricted class of simple systems: therefore, several approximate methods, with a different degree of complexity and accuracy, have been proposed. Perhaps, the most popular approaches are the methods based on Gaussian and non-Gaussian closure schemes and on approximate solutions of the Fokker–Planck–Kolmogorov (FPK) equation, which are well codified in the literature (e.g. [1–5]). Of course, different approaches are also available (e.g. the methods based on the maximum entropy principle and on the dissipation energy balancing) and, among these, the stochastic averaging (SA) method [6,7] is applied in this paper in order to estimate the mean upcrossing rate and the power spectral density (PSD) of the response of a single-degree-of-freedom (SDoF) oscillator with non-linear restoring force under white noise input.

It is well known that the mere probabilistic characterization of the response process is not sufficient in a reliability analysis. In fact, under the assumption that a vibrating system fails as soon as the response firstly exits a given safe domain, the statistics of the first passage time have to be estimated, starting from the knowledge of the statistics of the response to the random excitation. This is recognized to be one of the most complicated problem in computational stochastic mechanics, and no exact solutions have been derived, even in the simplest case of SDoF linear oscillators under stationary white noise; hence, a number of approximate formulations are available in the literature.

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Among these, the most popular one is the so-called "Poisson approach" (e.g. [3]), in which the response upcrossings of a deterministic threshold are assumed to be statistically independent events. This classical approach, however, proves to be too conservative when the response process is narrowband, and/or when the threshold is not high enough. In these situations, in fact, consecutive upcrossings of the response process cannot be realistically considered as independent events, as they tend to occur in clumps, whose mean size depends on the spectral bandwidth of the response. The latter, then, has to be somehow accounted for in order to improve the results.

The Gaussian Censored Closure (GCC) technique proposed by Senthilnathan and Lutes [8] reveals the same bounds, since also in this case the clumping tendency of the response upcrossings is neglected. With the purpose of overcoming this drawback, Muscolino and Palmeri [9,10] recently introduced an expedient "censorship factor," which can be directly related to the spectral bandwidth of the response process; the use of the Gumbel model as "uncensored" PDF for the peak response, instead of the Gaussian one, further improves the results. Effectiveness, accuracy and computational advantages of this technique have been proved in the reliability analysis of linear structures, also in the general case of multi-degree-of-freedom systems subjected to coloured noises [11].

Aim of this paper is to extend the use of the proposed technique, termed advanced censored closure (ACC), to non-linear SDoF oscillators under stationary white noise. The results herein presented complement those included in Ref. [12], in which only the case of non-linear damping is coped with. At the best knowledge of the authors, these are the first "consistent" applications of a censored closure technique in the reliability analysis of non-linear dynamical systems. The only examples found in the literature, in fact, are the pioneering papers by Suzuki and Minai [13,14] in which, however, the response of elastoplastic structures is assumed to be Gaussian.

The proposed ACC technique is amply illustrated by numerical examples, which demonstrate the superiority with respect to Poisson approach and GCC technique, especially when the response process of the non-linear oscillator is narrowband.

2. Response analysis

Let us considerer the random vibration of a non-linear SDoF oscillator driven by a zero-mean stationary white noise W_t:

$$m\ddot{X}_t + f(X_t, \dot{X}_t) = W_t \tag{1}$$

where X_t is the random process that describes the motion, $t \geqslant 0$ is the generic time instant, m is the inertia, $f(x, \dot{x})$ is the non-linear restoring force, which depends on the instantaneous values of displacement, $X_t = x$, and velocity, $\dot{X}_t = \dot{x}$, and the over-dot denotes the time derivate. For the simplicity purpose, the restoring force is assumed to be symmetric with respect to the origin of the phase plane $\{x, \dot{x}\}$, i.e. $f(x, \dot{x}) = f(-x, -\dot{x})$. As a consequence, in our analyses the mean value of the response process X_t is zero.

From a probabilistic point of view, the state variables of the system, X_t and $\dot{X}_t$, are characterized in stationary conditions by the knowledge of the time-independent joint probability density function (PDF), $p_{X\dot{X}}(x, \dot{x})$. Given that $f(x, \dot{x})$ is a non-linear function, $p_{X\dot{X}}(x, \dot{x})$ is non-Gaussian, and as strong is the non-linearity in the reaction force, as largely the PDF of the response deviates from the Gaussianity. In these situations, when the exact solution is not available, the PDF of the response can be estimated via a number of approximate methods known to the literature.

Among others, the stochastic averaging (SA) method is widely adopted, being versatile and quite straightforward [6]. The method, herein applied in the form recently presented in Ref. [7], operates under the assumption that the motion is pseudo-harmonic, that is

$$X_t = A_t \cos[\omega_{\text{eff}}(A_t)t + \Phi_t]$$
$$\dot{X}_t = -A_t \sin[\omega_{\text{eff}}(A_t)t + \Phi_t]$$

in which the amplitude A_t and the phase Φ_t constitute a 2-variate random process "slowly" varying with respect to the time t, and $\omega_{\text{eff}}(a)$ is a deterministic function that describes the "effective" value of the amplitude-dependent circular frequency of vibration:

$$\omega_{\text{eff}}(a) = \sqrt{\frac{k_{\text{eff}}(a)}{m}}$$

For a given value of the amplitude, $A_t = a$, the method furnishes the effective stiffness, $k_{\text{eff}}(a)$, and the effective damping coefficient, $c_{\text{eff}}(a)$, as solution of the implicit equations:

$$k_{\text{eff}}(a) = \frac{1}{\pi a} I_{\text{c}}(a)$$
$$c_{\text{eff}}(a) = -\frac{1}{\pi \omega_{\text{eff}}(a) a} I_{\text{s}}(a)$$

where $I_{\text{c}}(a)$ and $I_{\text{s}}(a)$ are the integral functions associated with the in-phase and out-of-phase components of the restoring force, respectively:

$$I_{\text{c}}(a) = \int_0^{2\pi} f[a\cos(\theta), -a\omega_{\text{eff}}(a)\sin(\theta)]\cos(\theta)\,\mathrm{d}\theta$$
$$I_{\text{s}}(a) = \int_0^{2\pi} f[a\cos(\theta), -a\omega_{\text{eff}}(a)\sin(\theta)]\sin(\theta)\,\mathrm{d}\theta$$

In stationary conditions, the Rayleigh-like approximate PDF of the amplitude can be evaluated once the functions $\omega_{\text{eff}}(a)$ and $c_{\text{eff}}(a)$ are known:

$$p_A(a) = \frac{1}{N_A}\frac{m\omega_{\text{eff}}(a)a}{\sqrt{\pi S_0}}\exp\left[-\frac{m\Pi_A(a)}{\pi S_0}\right] \tag{2}$$

where S_0 is the level of the uniform PSD of the white noise input, and N_A is just a normalization constant, which can be computed by satisfying the axiomatic condition: