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Explaining extreme waves by a theory of stochastic wave groups

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Abstract

It is well known that in a Gaussian sea an extreme wave event is a particular realization of the space–time evolution of a well defined linear wave group, in agreement with the theory of quasi-determinism of Boccotti [Boccotti P. On mechanics of irregular gravity waves. Atti Acc Naz Lincei, Memorie 1989;19:11–170] and the Slepian model of Lindgren [Kac M, Slepian D. Large excursions of Gaussian processes. Ann Math Statist 1959;30:1215–28; Lindgren G. Some properties of a normal process near a local maximum. Ann Math Statist 1970;4(6):1870–83]. In this paper, the concept of stochastic wave groups is proposed to explain the occurrence of extreme waves in nonlinear random seas, according to the dynamics imposed by the Zakharov equation [Zakharov VE. Statistical theory of gravity and capillary waves on the surface of a finite-depth fluid. J Eur Mech B—Fluids 1999;18(3):327–44]. As a corollary, a new analytical solution for the probability of exceedance of the crest-to-trough height is derived for the prediction of extreme wave events in nonlinearly modulated long-crested narrow-band seas. Furthermore, a generalization of the Tayfun distribution [Tayfun MA. On narrow-band representation of ocean waves. Part I: Theory. J Geophys Res 1986;91(C6):7743–52] for the wave crest height is also provided. The new analytical distributions explain qualitatively well recent experimental results of Onorato et al. [Onorato M, Osborne AR, Cavaleri L, Brandini C, Stansberg CT. Observation of strongly non-Gaussian statistics for random sea surface gravity waves in wave flume experiments. Phys Rev E 2004;70:067302] and the numerical simulations of Socquet-Juglard et al. [Socquet-Juglard H, Dysthe K, Trulsen K, Krogstad HE, Liu J. Probability distributions of surface gravity waves during spectral changes. J Fluid Mech 2005;542:195–216].

Keywords: Extreme crest; Zakharov equation; Benjamin–Feir instability; Probability of exceedance; Freak wave; Quasi-determinism

1. Introduction

Freak waves are extraordinarily larger water waves with potentially devastating effects on offshore structures and ships. The freak event occurred on January 1st 1995 under the Draupner platform in the North Sea [8] provides evidence that such waves can occur in the open ocean. During this freak event, an extreme crest with an amplitude of 18.5 m occurred. The maximal wave height of 25.6 m was much more than twice the significant wave height of about 10.8 m. Up to date, it is still unclear what are the basic physical mechanisms that can produce such a energy focussing in a small area of the ocean. Recent studies have outlined two relevant scenarios for the occurrence of freak waves.

The first scenario is based on the nonlinear mechanism of second order bound waves, i.e. waves that do not satisfy the linear dispersion relation. They can cause a concentration of wave energy in a small area of the ocean through the time–space focusing of a second order nonlinear wave group as explained by Fedele [9] and Fedele and Arena [10] by means of the theory of quasi-determinism of Boccotti [1,11]. In this model the wave crest amplitude H_c is distributed according to the Tayfun distribution [5] whereas the second order crest-to-trough height still follows the Rayleigh distribution with good approximation. Thus, a freak event, that is a wave for which $2H_c/H_s > 2.2$, is a rare realization of a second order nonlinear wave population, H_s being the significant wave height. Moreover

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the underlying stochastic process is non-Gaussian but stationary and ergodic.

The second scenario is based on the third order four-wave resonance interaction of free waves, i.e. waves that do satisfy the linear dispersion relation [12–14]. In this case, the weakly nonlinear energy transfer among resonant quartets occurs according to the deterministic Zakharov integral differential equation [4,15]. For the case of long-crested narrow-band waves, the Zakharov equation reduces to the nonlinear Schrödinger (NLS) equation [14] valid for narrow-band spectra or to the enhanced NLS equation derived by Dysthe [16], valid for broader spectral bandwidth and larger steepness. In this case, an initial wave packet changes in time when the energy flows from the central mode to the side-band modes because of the Benjamin–Feir instability [17]. If only a discrete but finite set of side-band modes are considered and the discretization of the NLS equation is done consistently to maintain its Hamiltonian integrability, then the energy eventually flows back to the central mode restoring the wave to its initial state. This energy exchange occurs in time recurrently and it produces an effect of intermittence to the surface displacement: high crests occur intermittently in time, affecting the statistics of the wave crests which tends to deviate from being Gaussian as described by Janssen [14]. Extreme events become more probable due to the Fermi–Pasta Ulam recurrence and the kurtosis of the wave distribution increases [18,19]. In the limit of an infinite set of side-band modes (continuous spectrum) the recurrence phenomenon is suppressed by phase mixing and the spectrum asymptotically relaxes toward a statistical non-Gaussian steady state [7]. According to this model a freak wave is thus a typical realization of a special wave population.

In recent wave tank experiments, Onorato et al. [6,20] show that a Benjamin–Feir type modulation instability is dominant only in long-crested narrow-band waves. In order to characterize the nonlinear behavior of the random field, Onorato et al. [18] defined a kind of 'Ursell number' as the ratio between the nonlinear and dispersive terms of the NLS equation, which is the Benjamin–Feir index (BFI) introduced by Janssen [14], that is

$$\mathrm{BFI} = \frac{\sqrt{2}\varepsilon_d}{2\Delta K/k_d}$$

Here, ε_d is the characteristic steepness of the linear waves, k_d is the wave number corresponding to the peak of the linear spectrum and $2\Delta K$ is the bandwidth of the wave spectrum. Onorato et al. [20] investigated the spatial evolution of quasi-stationary Gaussian initial conditions generated by a wavemaker and found that the kurtosis tends to exceed its Gaussian value and stabilizes monotonically as the distance from the wavemaker increases. They found that larger BFI values yield stronger deviations from the Gaussian statistics. Further, the deviation from Gaussianity strongly affects the wave-crest amplitudes whose sample distribution derived from the wave tank measurements seems to deviate from the Tayfun distribution [5]. Strong deviations from the Rayleigh law were also found for the crest-to-trough height distribution [6]. Socquet-Juglard et al. [7] arrive at the same conclusions by studying the time evolution of homogenous random fields by means of numerical simulations.

Both the experimental results of Onorato et al. [6,20] and the numerical simulations of Socquet-Juglard et al. [7] show also that for the case of multidirectional random waves, the nonlinear effects are due dominantly to bound waves and the Tayfun distribution explains very well the crest statistics. However, no analytical models like the Tayfun distribution are currently available for the case of third order nonlinear narrow-band waves, that can be strongly non-Gaussian.

In this paper, firstly the theory of wave groups in a Gaussian sea is revisited in the context of the theory of quasi-determinism of Boccotti [11]. It is shown that the linear wave group can be thought as a '*gene*' of a Gaussian sea when the interest is in the dynamics of the process at high energy levels. Extreme events most likely occur during the dynamics of a single wave group.

In the second part of the paper, guided by the theory of quasi-determinism of Boccotti [1,11] and supported by the analytical work of Lindgren [3,21] and the regression approximation method of Lindgren and Rychlik [22,23], a stochastic theory of wave groups is presented to explains the occurrence of extreme waves in nonlinear random seas. As a corollary, a new probability of exceedance of the crest-to-trough height that takes in to account the quasi-resonance interaction in long-crested narrow-band seas is derived. The theory presented here is easily extended to consider second order bound wave nonlinearities, thus providing a generalization of the Tayfun distribution for the wave crest height [5]. Finally, comparisons with recent wave tank experimental results [20,6] are presented.

2. Wave groups in a Gaussian sea

In the context of Gaussian waves, in the early 1970s Lindgren proved that in the time domain, locally to a very high crest, the surface displacement tends to assume the shape of the autocovariance function $\psi(T) = \langle\eta(t)\eta(t+T)\rangle$ where $\langle\cdot\rangle$ is the time average operator [3,21]. Tromans et al. [24] applied this time-domain formulation for the first time to offshore applications by the name of 'New Wave theory'. Extreme wave heights in narrow-band Gaussian seas are distributed according to the Rayleigh distribution as shown by Longuet-Higgins [25]. Because of the symmetry of the Gaussian process, both crest and trough distributions follow the same Rayleigh law for narrow-band spectra. For more general Gaussian processes with finite-band spectra, it is well known that the Rayleigh distribution is an upper bound for the probability of exceedance of the both crest heights and crest-to-trough wave heights. As regard to the crest height, the Rayleigh law tends to be asymptotically exact in the limit of large crest amplitudes. Higher