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Interval and fuzzy dynamic analysis of finite element models with superelements

Hilde De Gersem *, David Moens, Wim Desmet, Dirk Vandepitte

Katholieke Universiteit Leuven, Department of Mechanical Engineering, PMA, Celestijnenlaan 300B, B-3001 Heverlee, Belgium

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Abstract

This paper uses the interval and fuzzy finite element method for the eigenvalue and frequency response function analysis of structures with uncertain parameters. In order to reduce the calculation time of the interval and fuzzy analyses, these non-probabilistic methods are combined with the component mode synthesis technique. Special attention is paid to the effect of uncertainties on the mathematical description of this substructuring technique. All concepts are illustrated through a benchmark structure example.

Keywords: Uncertainty; Interval finite element method; Fuzzy finite element method; Eigenvalues; Frequency response function; Component mode synthesis; Substructuring

1. Introduction

During the last decades, the exponential growth of computational power has enabled the finite element (FE) analysis of very complex and detailed numerical models of new designs and structures. However, it is often very difficult to define a reliable finite element model for numerical simulation and validation, as some numerical properties may be uncertain, e.g. due to manufacturing tolerances, variability on material characteristics, modelling simplifications, etc. Especially during the design of a product, numerical descriptions of model details are often not available or only partially known. Due to the ever growing demands imposed on new products, it is of utmost importance for design engineers to investigate the influence of all relevant uncertainties on the static and/or dynamic behaviour of structures. Hence, non-deterministic approaches are gaining interest in design and development areas. Nowadays, these techniques are applied in enhanced numerical tools as reliability assessment, robustness analysis and design optimisation.

The probabilistic concept is already well established for the extension of the deterministic finite element method towards uncertainty assessment. This has led to a number of probabilistic finite element procedures [1], that are often used in conjunction with Monte Carlo Simulations. Probabilistic methods are especially suitable in case of model variabilities for which information on both the range and the probability density function is available. However, the use of a probabilistic approach can lead to subjective results if the amount of statistical data on the uncertainties is limited and approximations and subjective information need to be included [2].

The fuzzy set theory, introduced by Zadeh [3] in 1965, provides a concept for the description of linguistic or subjective knowledge and incomplete data in a non-probabilistic manner. The use of Zadeh's concept in the finite element context has led to the development of the fuzzy finite element method [4]. Its aim is to calculate the membership function of an output quantity, based on the fuzzy description of the uncertain input parameters. By using the α-level technique, the interval finite element method (IFEM) forms the core of the fuzzy procedure.

Over the last decade, several interval FE procedures have been reported, such as the vertex method, the

* Corresponding author. Tel.: +32 16 328606; fax: +32 16 322987.
E-mail address: hilde.degersem@mech.kuleuven.be (H. De Gersem).

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global optimisation approach and the interval arithmetic approach. A hybrid procedure for the interval frequency response function (FRF) calculation has been developed and implemented by Moens and Vandepitte [5,6]. In this approach, a global optimisation step determines the correct intervals on the modal parameters, which are then combined to calculate the FRF upper and lower bounds with an interval arithmetic procedure. Special attention is paid to the limitation of the conservatism and the computational efficiency of the method. The hybrid FRF procedure was first developed for undamped and proportionally damped structures [7], and it is recently extended for fuzzy modal damping parameters [8]. Various numerical case studies prove that the interval and fuzzy finite element method provide useful tools to perform early design validation and optimisation [9].

For industrial size models with a large number of uncertainties, the computation time of the interval and the fuzzy finite element method can be considerable. A reduction in calculation time can be achieved by the substructuring of large models into superelements, which are then independently processed and reduced [10]. Subsequently all reduced superelements are recombined to form the reduced structural system for which the desired output quantities are calculated. However, the introduction of uncertainties in a substructuring technique requires a concept for the description of uncertainty on the superelement level. In this paper, the fuzzy finite element method is combined with the Craig–Bampton component mode synthesis method, in which the static and dynamic behaviour of each superelement are represented by a set of component modes.

This paper describes the basic properties of the fuzzy finite element method in Section 2. This section also gives an overview of the hybrid procedure for fuzzy frequency response function analysis, as developed by Moens. Section 3 describes the basic principles of the component mode synthesis technique. In Section 4, the above mentioned methods are combined for the interval and fuzzy eigenfrequency and frequency response function analysis of uncertain FE models with superelements. Different approaches to handle uncertainties in superelements are proposed. In Section 5, interval and fuzzy dynamic analyses are performed on two superelement FE models of the Garteur benchmark aircraft. The accuracy and numerical efficiency of the proposed approaches for superelement uncertainty handling are compared.

2. The fuzzy finite element method

2.1. Fuzzy sets

The concept of fuzzy sets, introduced by Zadeh [3] in 1965, has gained an increasing popularity during the last two decades. Its most important property is that it is capable of describing linguistic and therefore incomplete information in a non-probabilistic manner. Whereas a classical set clearly distinguishes between members and non-members, a fuzzy set introduces a degree of membership, represented by the *membership function*. For a fuzzy set $\tilde{x}$, the membership function $\mu_{\tilde{x}}(x)$ describes the grade of membership to the fuzzy set for each element x in the domain X:

$$\tilde{x} = \{(x, \mu_{\tilde{x}}(x)) | (x \in X)(\mu_{\tilde{x}}(x) \in [0, 1])\} \tag{1}$$

If $\mu_{\tilde{x}}(x) = 1$, x is definitely a member of the set $\tilde{x}$, whereas if $\mu_{\tilde{x}}(x) = 0$, x is definitely not a member of the set $\tilde{x}$. For all x with $0 < \mu_{\tilde{x}}(x) < 1$, the membership is not certain. The most frequently applied membership function shapes are the triangular and Gaussian shape.

2.2. Fuzzy numerical analysis

The fuzzy finite element method (FFEM) aims to obtain a fuzzy description of an FE analysis result, starting from the fuzzy descriptions of all non-deterministic FE model parameters. In practice, the description of uncertain parameters and quantities using fuzzy sets can be implemented using the *α-level strategy*. This approach subdivides the membership function range into a number of α-levels. The intersection with the membership function of the input uncertainties at each α-level results in an interval $x^I_{i,\alpha} = [\underline{x}_i, \bar{x}_i]_\alpha$. With these input intervals of the α-sublevel, an interval finite element (IFE) analysis is performed, resulting in an interval for the analysis result at the considered α-level. Finally, the fuzzy solution is assembled from the resulting intervals at each sublevel.

The fuzzy finite element technique consists of the application of the α-level strategy on the numerical procedure of the deterministic FE analysis. Through this procedure, the fuzzy FE analysis can be interpreted as a large-scale sensitivity analysis, which can be used to study the combined effect of the interval bounds of design variables on critical design properties. See [11] for a more comprehensive overview of fuzzy analysis for non-deterministic analysis in engineering design.

Through the α-level strategy, the interval problem corresponding to the considered FE analysis forms the core of the fuzzy procedure. A number of general interval solutions for the interval FE problem have been described in literature. The most popular one is the interval arithmetic approach, in which all basic deterministic algebraic operations are replaced by their interval arithmetic counterparts. Although this approach is computationally very inexpensive, it is of little practical use as it tends to largely overestimate the interval outcome of each operation, accumulating in a huge amount of conservatism in the interval analysis result of realistic problems. Therefore, other approaches such as the vertex method [12], the transformation method [13,14] and the global optimisation method [4] have been proposed for the solution of the underlying interval FE problem. Each of these techniques has its specific disadvantages when applied to realistical engineering problems, such that none of them has been established as standard interval procedure, and the choice of technique depends on the type of the conducted FE