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ScienceDirect

Computers and Structures 85 (2007) 320–329

Computers & Structures

www.elsevier.com/locate/compstruc

Bounds on structural system reliability in the presence of interval variables

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Received 15 October 2005; accepted 31 October 2006

Abstract

The failure of a structural system is usually governed by multiple failure criteria, all of which are to be taken into consideration for reliability estimation. If all the uncertain parameters are defined as random variables, then the system reliability can be estimated accurately by using existing techniques. However, when modeling variables with limited information as intervals with upper and lower bounds, the entire range of these bounds should be explored while estimating the system reliability. The computational cost involved in estimating reliability bounds increases tremendously because a single reliability analysis, which is a computationally expensive procedure, is needed for each configuration of the interval variables. To reduce the computational cost involved, high quality function approximations for individual failure functions and the joint failure surface are considered in this paper. The accuracy and efficiency of the proposed technique are demonstrated with numerical examples.

Keywords: System reliability; Random variables; Uncertain intervals; Fast Fourier transforms; Convolution integral; Reliability bounds

1. Introduction

Uncertainties present in the design process need to be quantified and propagated to assess the safety of the design. Based on the information available, these uncertainties might be quantified as random or nonrandom variables. If sufficiently large amount of data about a particular variable is available, then its variation can be approximated by using a probability distribution. This random uncertainty can be propagated using existing probabilistic methods. But if the information about a particular variable is limited to a lower and upper bound, then its variation cannot be approximated using a probability distribution. The entire intervals are to be analyzed in estimating the response bounds. But in most problems, information might be available to represent some variables with a probability distribution while information about some variables might be sparse. Therefore, this paper focuses on problems with multiple failure criteria for which some uncertainties can only be quantified as intervals while some are random in nature.

A structure consists of many individual components, all of which have the potential to fail. Reliability analysis of structural systems involves evaluating this potential to failure, for various performance criteria or limit-states from different disciplines that might be correlated. Moreover, when dealing with systems where the uncertain parameters are modeled using both random and interval variables, every configuration of the interval variables is to be explored to determine the bounds on the reliability. Therefore, computational effort involved in estimating the failure probability increases tremendously in the presence of multiple limit-states and mixed uncertain variables. In the case of systems with infinite limit-state functions, only the limit-state functions which place an active role in the reliability estimation can be considered for determining the system reliability. These limit-states can be obtained by considering the active constraints in the initial design process.

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doi:10.1016/j.compstruc.2006.10.012

When dealing with only random variables, the failure probability of a structural system is obtained by solving the multi-dimensional integral

$$p_f = \int_{\Omega} f_X(\boldsymbol{X})\,\mathrm{d}\boldsymbol{X} \qquad (1)$$

where p_f is the probability of failure, $f_X(\boldsymbol{X})$ denotes the joint probability density function of the vector for the basic random variables, $\boldsymbol{X}$ and Ω is the joint failure region modeled by all of the limit-state functions. This multifold integral has to be solved multiple times to estimate the bounds on system reliability. In theory, the probability density function of a random variable which is a linear summation of independent random quantities can be obtained by convoluting the individual density functions [1]. Therefore, when the failure surface is expressed as a linear combination of random variables, Eq. (1) can be transformed into a convolution integral and evaluated by the use of fast Fourier transforms.

Monte Carlo simulation can be used to estimate the probability of failure numerically. However, this simulation involves tremendous computational cost due to the large number of exact function evaluations that are required, which come from computationally expensive finite element analysis (FEA) or computational fluid dynamics (CFD) simulations. Even if the cost is not a constraint, the error associated with randomly sampling design points for the simulation leads to inaccuracy in the results. This is because the random numbers generated using pseudo random number generators tend to form clusters and are not uniformly distributed over the entire design space [2]. Moreover, the accuracy of the estimated failure probability is also dependent on the number of samples and seed number used in the Monte Carlo simulation. For aerospace applications where the probability of the entire system is expected to be 10^{-7}, the structural subsystem failure probability will be of the order of 10^{-10}, requiring at least 10^{11} samples to perform crude Monte Carlo simulation. For a case where 100 evaluations of an approximate limit-state can be performed in 1 s, we would need 10^9 s to run Monte Carlo which would require about 31.7 years. Therefore, there is a clear need to develop probability integration schemes that are applicable for these classes of problems.

To reduce the computational cost involved, researchers [3–5] have explored the use of surrogate representations of the failure surface to compute the failure probability. Most of these methods were developed for handling system models with only random variables. While algorithms that deal with mixed variable problems [6,7] exist, they do not have any provision for handling multiple limit-state functions. Therefore, methods need to be developed for systems that are modeled using mixed variables and have multiple failure criteria.

The use of fast Fourier transforms (FFT) for solving the convolution integral has been explored previously [8,9] using surrogate models for representing the failure surface. This surrogate model has to be a separable function that can be linearized in order to be able to apply the method. This approach has been applied for various problems to determine the failure probability of single limit-state function. However, when dealing with multiple limit-states, the failure surface to be approximated is often highly nonlinear as it is comprised of the intersection of all the failure modes. For this highly nonlinear surface, a single approximation will not be sufficiently accurate. Therefore, in this paper, a methodology [10] is used that evaluates the convolution integral based on several approximations, each of which is accurate in a certain region of the entire design space. This methodology is discussed in the next section using a numerical example with two random variables and extended to multiple random variables.

When all the variables are defined as intervals, then interval analysis techniques can be used to estimate the lower and upper bounds on the response. Interval arithmetic provides an exact bound if all the variables occur only once in the function. This problem of dependency [11] estimates a wider bound for the response if a variable occurs more than once. Interval uncertainties can also be propagated through the structure by including them in the finite element formulation [12]. A static structural problem can be expressed in the form of a system of linear interval equations which are solved to obtain the bounds on the structural response. In the presence of closed-form expressions for the limit-state functions, the interval variables can be transformed so that each variable appears only once in the expression thereby providing an exact estimate of the bounds. But when dealing with implicit limit-state functions, this technique is not feasible due to the absence of a closed-form relation between the response and the interval variables. So when dealing with implicit limit-state functions, the entire intervals are to be explored to obtain the configurations that result in the bounds of the response.

In the presence of both random and interval variables, every combination of the interval variables has an unknown probability. Therefore, when determining the bounds on reliability, the entire bounds of the interval variables are to be explored. Moreover, the joint failure surface changes based on the configuration of the interval variables. One approach for dealing with interval variables is to assume a uniform distribution for the variables. By doing this, additional information is added about these variables resulting in an inaccurate estimate of the reliability. Therefore, interval uncertainty has to be handled by analyzing the whole interval to estimate the bounds. In this paper, a methodology is presented for estimating the system reliability bounds for problems where some of the uncertain parameters are random in nature and some are defined as intervals. To facilitate the exploration of the intervals and modeling of a new joint failure surface for every combination of the interval variables, the implicit limit-state functions are modeled using high quality approximations. Once the joint failure surface is modeled, fast Fourier transforms are used to solve the convolution integral accurately.