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Efficient strategies for reliability-based optimization involving non-linear, dynamical structures

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ABSTRACT

This contribution proposes a framework for performing reliability-based optimization efficiently. The proposed approach, based on a decoupling approach and sequential approximations, introduces an efficient means for estimating reliability sensitivity along with the application of a line search strategy and weighted approximations. Two examples involving non-linear structures subject to dynamic loading are presented, showing the advantages and efficiency of the proposed framework.

1. Introduction

The aim of engineering methods is designing systems and structures which fulfill most economically certain predefined performance objectives. This can be accomplished by means of, e.g. optimization tools [1,17]. The application of optimization procedures allows minimizing overall costs, while ensuring that prescribed constraints (referring to, e.g. structural performance) are satisfied.

Optimization procedures offer a sound basis for structural design. However, it should be noted that in realistic situations, it is impossible to characterize structural performance deterministically, as many parameters (such as loadings, geometrical properties and material properties) are of uncertain nature. These uncertainties can dramatically affect the performance of a system; thus, they must be explicitly accounted for during the optimization procedure [10,11,14]. Reliability-based optimization (RBO) offers an appropriate framework for design under uncertainties, as it incorporates reliability measures within the optimization problem (see, e.g. [8,13,15,23,32,37,39,46,50]).

Recent developments in different fields have allowed the application of RBO in a number of challenging problems. For example, the application of advanced simulation techniques allows estimating reliability of involved structural systems (see, e.g. [3,5,27,28,42,43]). Specialized strategies for metamodeling (see, e.g. [19,36,37]), efficient sensitivity analysis (see, e.g. [2,29,33,39,49]), stochastic search algorithms (see, e.g. [45,46]) and specific approximation strategies (see, e.g. [8,15,22,24]) also provide means for solving RBO problems efficiently.

In spite of the developments described above, the application of RBO in problems of engineering interest has still remained limited due to high numerical costs. Therefore, the aim of this contribution is proposing a most efficient approach for RBO, with emphasis on a particular class of problems, namely minimization of the structural weight under a reliability constraint referring to the first excursion probability of non-linear structures subject to stochastic dynamic excitation. The proposed approach is based on the concept of decoupling (see, e.g. [15]) and sequential approximations (see, e.g. [17,20]), i.e. separation of reliability analysis and the optimization procedure by means of explicit approximations of the reliability as a function of the design variables. The innovative part of this contribution is in the application of decoupling and sequential approximations in combination with three specific strategies:

1. *Efficient reliability sensitivity estimation.* Reliability sensitivity (i.e. derivative of reliability with respect to design variables) can be used for constructing an approximate representation of the failure probability as an explicit function of the design variables. In order to estimate such sensitivities, it is proposed to use an approximate model based on a linearization of the performance function (associated with the reliability problem) in the space of the design variables. The most salient feature of the proposed approach is that it requires a single reliability analysis plus some additional structural analyses in order to estimate reliability sensitivity.

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2. *Line search*. Line search is a tool used customarily in non-linear programming methods [6], as it allows exploring the design variable space in a simplified manner (in 1 dimension). In this contribution, a line search strategy is applied for producing feasible design solutions (i.e. a design which fulfills a reliability constraint) efficiently.
3. *Weighted approximation of the failure probability*. Reliability estimates computed using simulation techniques have an inherent variability. It has been observed that, in some cases, such variability may prevent convergence during optimization. For coping with this problem, a weighting strategy is introduced; the basic concept of this strategy is calculating a weighted average approximation of structural reliabilities.

The structure of this contribution is as follows. In Section 2, the formulation of the RBO problem is presented. Section 3 addresses the framework for solving RBO problems using the decoupling approach and the sequential approximate optimization (SAO) algorithm. Section 4 addresses the issue of reliability sensitivity estimation. The application of a line search strategy and weighted approximation of the failure probability is discussed in Section 5. Two application examples showing the advantages of the proposed strategies are presented in Section 6.

2. Definition of the RBO problem

2.1. Basic formulation

It is assumed that the uncertain parameters associated with a particular structural system are grouped in a n_θ-dimensional vector $\boldsymbol{\theta}$ ($\boldsymbol{\theta} \in \Omega_\theta$) and that their relative plausibility is described by means of a joint probability density function ($f(\boldsymbol{\theta})$). Moreover, it is assumed that the design variables of the problem are deterministic quantities, grouped in a n_x-dimensional vector $\boldsymbol{x}$ ($\boldsymbol{x} \in \Omega_x$).

In this contribution, the RBO problem is formulated as the minimization of the weight of a structure (or other deterministic functions) subject to probabilistic constraint(s) (related with the structural performance) and deterministic constraint(s). In mathematical terms, this is:

$$\begin{aligned} &min \quad W(\boldsymbol{x}), \quad \boldsymbol{x} \in \Omega_x \\ &\text{subject to} \\ &p_{F,i}(\boldsymbol{x}) \leqslant p_{F,i}^{tol}, \quad i = 1, \ldots, n_{PC} \\ &h_j(\boldsymbol{x}) \leqslant 0, \quad j = 1, \ldots, n_{DC} \end{aligned} \tag{1}$$

where $W(\cdot)$ is the structural weight, $p_{F,i}(\cdot)$ represents the probability of failure with respect to the ith performance criterion (which should be smaller than a certain tolerable margin, $p_{F,i}^{tol}$) and $h_j(\cdot)$ is the jth deterministic constraint.

For the sake of simplicity, in this work only a single probabilistic constraint is considered, i.e. $n_{PC} = 1$. This assumption does not imply loss of generality, as the proposed approach can be extended to more probabilistic constraints; in such case, the procedure proposed in this contribution for constructing an approximate representation of the failure probability should be repeated a total of n_{PC} times. In addition, it is assumed that $\boldsymbol{x}$ is a real-valued, deterministic vector. It should be noted that the formulation of the RBO problem posed in Eq. (1) is certainly a simplification, as it ignores important aspects of a real engineering problem such as partial damage states [30], i.e. loss of serviceability of a structural system. Nonetheless, the formulation of the RBO problem in Eq. (1) suffices for the purposes of this work, as the methodology developed in this paper could be (in principle) extended to other more general formulations.

2.2. Reliability assessment

For generating a metric of structural reliability, it is necessary to define the so-called *performance function* (usually denoted by g). This function depends on the design variables and uncertain parameters, i.e. $g = g(\boldsymbol{x}, \boldsymbol{\theta})$, and it should be defined such that $g(\boldsymbol{x}, \boldsymbol{\theta}) \leqslant 0$ whenever a particular set of the vectors $\boldsymbol{x}$ and $\boldsymbol{\theta}$ leads to an unacceptable structural response. In this contribution, the focus is on reliability constraints referring to the first excursion probability of non-linear structures subject to stochastic dynamic excitation, i.e. the probability that the structural response exceeds a prescribed threshold level within the duration T of a stochastic excitation. Assuming a time discretization t_z, $z = 1, \ldots, n_T$ (where Δt is the time discretization step, such that $(n_T - 1)\Delta t = T$), the performance function for this class of problems can be formulated as follows [3]:

$$g(\boldsymbol{x}, \boldsymbol{\theta}) = b - \max_{q=1,\ldots,n_R} \left(\max_{z=1,\ldots,n_T} \left(\frac{r_q(t_z, \boldsymbol{x}, \boldsymbol{\theta})}{r_q^*} \right) \right) \tag{2}$$

where $r_q(\cdot,\cdot,\cdot)$, $q = 1, \ldots, n_R$, denotes the qth structural response (e.g. absolute value of displacement, von Mises stress, etc.), r_q^*, $q = 1, \ldots, n_r$, denotes a threshold level for the qth structural response and b is a *normalized* threshold level; it is assumed that $r_q(\cdot,\cdot,\cdot)$ and r_q^* are defined in such way that their quotient is always equal or larger than zero. It is most interesting to note that the quotient $r_q(t_z, \boldsymbol{x}, \boldsymbol{\theta})/r_q^*$ can be interpreted as a *demand to capacity ratio*, as it compares the value of a structural response ($r_q(t_z, \boldsymbol{x}, \boldsymbol{\theta})$) with the maximum acceptable value of this response (r_q^*). Thus, introducing the concept of normalized demand $D(\boldsymbol{x}, \boldsymbol{\theta})$:

$$D(\boldsymbol{x}, \boldsymbol{\theta}) = \max_{q=1,\ldots,n_R} \left(\max_{z=1,\ldots,n_T} \left(\frac{r_q(t_z, \boldsymbol{x}, \boldsymbol{\theta})}{r_q^*} \right) \right) \tag{3}$$

the definition of performance function can be further simplified to:

$$g(\boldsymbol{x}, \boldsymbol{\theta}) = b - D(\boldsymbol{x}, \boldsymbol{\theta}) \tag{4}$$

It should be noted that the performance function and the normalized demand (Eqs. (2) and (3), respectively) associated with a first passage problem can be non-smooth [26]. However, numerical validations have shown that such property does not impose restrictions on the approach presented in this contribution and the examples analyzed.

In reliability analysis, the normalized threshold level (b) is usually set equal to one (i.e. $b = 1$), in order to ensure that the performance function is smaller than zero when a structural response exceeds its prescribed threshold level (i.e. $D(\boldsymbol{x}, \boldsymbol{\theta}) > 1$). Thus, throughout this contribution, it is assumed that a performance function has always associated a normalized threshold level equal to 1.

Once $g(\boldsymbol{x}, \boldsymbol{\theta})$ has been defined, the failure probability of a structural system can be defined as:

$$p_F(\boldsymbol{x}) = P[b - D(\boldsymbol{x}, \boldsymbol{\theta}) \leqslant 0 : b = 1] = P[g(\boldsymbol{x}, \boldsymbol{\theta}) \leqslant 0] \tag{5}$$

where $P[\cdot]$ denotes probability of occurrence. Alternatively, the failure probability can be defined by means of the following multidimensional integral:

$$p_F(\boldsymbol{x}) = \int_{g(\boldsymbol{x}, \boldsymbol{\theta}) \leqslant 0} f(\boldsymbol{\theta}) d\boldsymbol{\theta} \tag{6}$$

The integral in Eq. (6) can be evaluated by means of *simulation methods* [43]. Among different available simulation methods, Monte Carlo simulation (MCS) [35,47] is the most general technique. However, MCS is numerically demanding for estimating low failure probabilities (which are typical in engineering applications). In order to circumvent this issue, *advanced simulation techniques* have