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Shape sensitivities in the reliability analysis of nonlinear frame structures

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Abstract

A unified and comprehensive treatment of shape sensitivity that includes variations in the nodal coordinates, member cross-section properties, and global shape parameters of inelastic frame structures is presented. A novelty is the consideration of geometric uncertainty in both the displacement- and force-based finite element formulations of nonlinear beam-column behavior. The shape sensitivity equations enable a comprehensive investigation of the relative influence of uncertain geometrical imperfections on structural reliability assessments. For this purpose, finite element reliability analyses are employed with sophisticated structural models, from which importance measures are available. The unified approach presented herein is based on the direct differentiation method and includes variations in the equilibrium and compatibility relationships of frame finite elements, as well as the member cross-section geometry, in order to obtain complete shape sensitivity equations. The analytical shape sensitivity equations are implemented in the OpenSees software framework. Numerical examples involving a steel structure and a reinforced concrete structure confirm that geometrical imperfections may have a significant impact on structural reliability assessments.

Keywords: Shape sensitivity; Direct differentiation method; Geometrical imperfections; Structural reliability; Beam-columns; Nonlinear analysis; OpenSees

1. Introduction

A number of applications in structural engineering require the computation of the gradient of structural response quantities with respect to input parameters. This is referred to as response sensitivity analysis. The most common applications of response sensitivity analysis are to optimization problems, such as the minimization of structural cost subject to constraints and minimization of the difference between measured and numerical response for system identification purposes. Yet another optimization problem is posed in structural reliability analysis by the first and second-order reliability methods (FORM and SORM). These methods rely upon the determination of the "most probable failure point", which is the solution to a constrained optimization problem in the space of random variables. As a by-product, FORM analysis provides importance measures to rank the uncertain parameters according to their relative influence on the structural reliability. Importance measures remedy the problem that individual response sensitivities cannot be compared directly due to differing units. It is also emphasized that response sensitivities are useful as a stand alone product in structural design because they indicate the sensitivity of a structural response quantity to changes in the design parameters.

Three requirements are put forward by the application of response sensitivities in gradient-based optimization algorithms: efficiency, accuracy, and consistency. Efficient

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computation of response sensitivities is required to make gradient-based algorithms competitive with gradient-free methods, including response surface methods, in terms of computational cost. This is particularly important when the optimization is performed in a high-dimensional space of variables, in which case repeated runs to obtain gradients by finite differences is infeasible. Accuracy is imperative to avoid convergence problems in the optimization algorithms, which tend to perform poorly in the presence of even small inaccuracies in the gradients. The consistency requirement stems from the utilization of approximate numerical models to obtain the structural response. Because it is the approximate response that is employed in the optimization problem, it is the gradient of the approximate response that is required. Consequently, it is not of interest to pursue the ''exact'' gradient of the theoretical boundary value problem. In fact, this would lead to inconsistency between the function value and its gradient.

Two approaches are available to obtain response sensitivities: finite difference methods (FDMs) and the direct differentiation method (DDM). The finite difference approach employs re-runs of the structural analysis with perturbed parameter values to estimate the response sensitivity. As a result, it is a computationally inefficient approach. Moreover, FDMs suffer from accuracy concerns. It is a nontrivial task to select the value of the parameter perturbation for nonlinear problems. If the perturbation is too small, round-off errors are introduced; while if the perturbation is too large, local nonlinearities may lead to inaccurate estimates of the sensitivity. The consistency requirement, however, is satisfied by the FDMs because it is the approximate response that is employed in the finite difference equations.

The DDM provides an attractive alternative to FDMs. At the one-time cost of deriving and implementing analytical sensitivity equations within the finite element response algorithm, efficient, accurate, and consistent response sensitivities are obtained. No finite difference computations take place within the DDM; instead, the response equations are analytically differentiated and implemented on the computer alongside the ordinary response computations. A number of researchers have contributed to the development of such analytical equations, including Choi and Santos [2], Tsay and Arora [24], Liu and Der Kiureghian [15], Zhang and Der Kiureghian [25], Kleiber et al. [13], Conte et al. [3], Roth and Grigoriu [20], Scott et al. [22], and Haukaas and Der Kiureghian [9]. The DDM is more efficient than FDMs because repeated runs of the response analysis are unnecessary. Accuracy is ensured at the same precision as the response because the same equation solver is employed to obtain both the response and the response sensitivity. Consistency is achieved by differentiating the response equations after they have been spatially and temporally discretized by the finite element procedures. The DDM is thus the preferred approach to computing response sensitivities.

In this paper, the OpenSees software framework [16] is extended and applied for shape sensitivity analysis. OpenSees (open system for earthquake engineering simulation) is an open-source, object-oriented, general-purpose finite element code specifically developed for earthquake engineering analysis. OpenSees began as the computational platform for testbed simulations in the Pacific Earthquake Engineering Research Center (PEER) and has since been adopted by the NSF-sponsored George E. Brown Jr. Network for Earthquake Engineering Simulation (NEES). Work by Haukaas and Der Kiureghian [8] extends OpenSees with response sensitivity and reliability analysis capabilities which allow the analyst to characterize input parameters as random variables and compute probabilities of structural response events. This is termed finite element reliability analysis (FERA), which differs from so-called stochastic finite element methods that focus on second-moment statistics of the response. In contrast, reliability analysis and specifically FERA is suited to compute probabilities of rare response events. This addresses the growing demand in performance-based engineering to assess structural behavior during rare events of intense loading in a probabilistic manner.

The response sensitivity implementations in OpenSees are based on the DDM. The implementations are divided into an overarching framework and object-specific implementations. The latter reflect the fact that OpenSees is organized into element, section, and material objects. The framework for sensitivity computations, as well as specific implementations for selected elements, sections, and materials, is already in place. This includes sensitivities with respect to nodal coordinates, which previous studies suggest may be an important source of uncertainty in structural reliability, particularly when nonlinear structural behavior is considered [9].

In this paper, the DDM shape sensitivity equations include response sensitivities with respect to: (1) nodal coordinates, (2) global structural or member shape parameters, and (3) the dimensions and details of fiber-discretized cross-sections. Of particular significance is the development of unified shape sensitivity equations for beam-column elements in both the displacement- and force-based formulations. Gradient computations that incorporate shape sensitivity at all levels (structure, element, and section) are presented and their implementation in OpenSees allows the inclusion of a wide range of uncertain geometrical imperfections in a reliability analysis. Two numerical examples involving a steel structure and a reinforced concrete structure provide insight into the importance of uncertain geometrical imperfections relative to other uncertain structural properties.

2. The application of response sensitivities in finite element reliability analysis

The need for response sensitivities in this paper stems from structural reliability analysis. To achieve accurate