Computers and Structures 89 (2011) 496–506

Contents lists available at ScienceDirect

Computers and Structures

journal homepage: www.elsevier.com/locate/compstruc

Dynamics of shallow shells with geometrical nonlinearity interacting with fluid

I.D. Breslavsky, E.A. Strel'nikova, K.V. Avramov *

A.N. Podgorny Institute for Mechanical Engineering Problems, National Academy of Sciences of Ukraine, 2/10 Dm. Pozharskogo St., 61046 Kharkiv, Ukraine

ARTICLE INFO

Article history:
Received 2 March 2010
Accepted 3 December 2010
Available online 6 January 2011

Keywords:
Shallow shell interacting with a fluid
Hypersingular integral equation
Nonlinear modes
Backbone curves
Stability of vibrations

ABSTRACT

The vibrations of the shallow shell with geometrical nonlinearity submerged in a fluid are considered. Interaction of the shell with a fluid is described by linear hypersingular integral equation, which is solved by the boundary element method. The vibrations of the shell are described by the nonlinear finite-degree-of-freedom system. The vibrations are studied by the Shaw–Pierre nonlinear modes.

1. Introduction

Shells structures interacting with a fluid describe the elements of missiles, energetic and transport equipments. Such structures frequently perform vibrations, which might lead to fatigue breaking. Therefore, scientists and engineers are interested in investigations of interaction of fluid with thin-walled structures. Interaction of the closed cylindrical shells with fluids is considered in most studies devoted to this question [1]. Shallow shells submerged in a fluid are given less attention. Lindholm et al. [2] studied experimentally free vibrations of a cantilever plate particularly submerged in a fluid. Using the theory of potential flow, Meyerhoff [3] determined added masses of a fluid on square plate. He describes a pressure difference of fluid using dipole singularity. Korobkin and Khabakhpasheva [4] and Wang et al. [5] analyzed floating elastic shells interacting with a fluid on one side. Linear vibrations of a cantilever plate particularly submerged in a fluid are analyzed by Ergin and Ugurlu [6]. Wet vibrations eigenmodes are determined as a linear combination of dry eigenmodes, which are calculated by the finite element method. The paper [7] is devoted to the method of frequencies calculations for hydraulic turbine blades submerged in a fluid. The calculations are performed by the finite element method. In general, finite element method is frequently used to analyze linear vibrations of plates submerged in a fluid [8,9]. Fu and Price [10] studied the dynamics of a cantilever plate fully or partially submerged in a fluid. They determined the general forces of a fluid acting on plate surfaces. The orthogonality of the eigenmodes of the shallow shell fully or partially submerged in a fluid is studied by Fang [11]. Tosh and Frendi [12] analyzed forced vibrations of a clamped plate with geometrical nonlinearity interacting with a fluid. It is shown, that, if the external force amplitude is increased, the periodic vibrations of the shell are transformed into chaotic ones. Green and Sader [13] analyzed the vibrations of a beam submerged in a fluid. It is assumed that a fluid is restricted by a rigid wall near the vibrating body. The authors conclude that the wall changes qualitatively the vibrations of the beam. Kantor and Strel'nikova [14] and Naumenko and Strel'nikova [15] considered different numerical and analytical methods for solutions of hypersingular integral equations. The linear plate vibrations in a fluid are described by these integral equations. Finite element analysis of fluid interacting with structures is considered by Bathe and his coauthors [16–21]. The choice of the finite elements to obtain the solutions with enough accuracy is discussed in [20,21].

The vibrations of the shallow shell with complex base submerged in a potential fluid are treated in this paper. It is assumed that the shell performs geometrical nonlinear deformations and a fluid is described by linear model. The thin shell is considered. Therefore, shear and rotational inertia are not taken into account. The influence of a fluid on a shell is described by the hypersingular integral equation, which is solved by the boundary element method. Nonlinear vibrations of the shells interacting with a fluid are expanded into truncated wet eigenmode series. Finite-degree-of-freedom model is derived. The Shaw–Pierre nonlinear normal modes are used to analyze the obtained dynamical system. The novelty of this paper is the following. The nonlinear vibrations of the flexible shallow shells with complex base and variable thickness interacting with a fluid are investigated.

The paper is organized in the following way. Section 2 is devoted to the analysis of linear vibrations of the shallow shells

* Corresponding author.
E-mail address: kvavr@kharkov.ua (K.V. Avramov).

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doi:10.1016/j.compstruc.2010.12.006

with variable thickness. Mathematical model of a fluid and method of linear vibrations analysis are treated in Section 3. The system of ordinary differential equations describing the nonlinear vibrations of the shallow shells interacting with a fluid is derived in Section 4. In the final sections the suggested approach is applied to analyze the nonlinear vibrations of hydraulic turbine blade.

2. Shell linear vibrations in vacuum

The vibrations of the shallow shells with complex base, variable thickness and arbitrary curvature are considered in this paper. The shallow shell is clamped on the part of the contour and free on the rest part. As thin shells are analyzed, rotary inertia and shear deformations are not taken into account. Fig. 1 shows the middle surface of the shell. Two orthogonal directions, which are lines of the principle curvatures, are taken on the shell middle surface. Curvilinear coordinates α and β are counted off along these lines (Fig. 1). The third axis z is directed orthogonal to the plane base.

The aim of this paper is analysis of vibrations of the shallow shell accounting geometrical nonlinearity. However, it is impossible to analyze nonlinear vibrations without a previous analysis of linear vibrations. Therefore, the linear vibrations of shell are treated in detail.

The boundary conditions on the clamped part of the shell are the following:

$$w = \frac{\partial w}{\partial n} = 0, \quad u = 0, \quad v = 0, \quad (\alpha, \beta) \in \partial \Lambda, \tag{1}$$

where $u(\alpha, \beta, t)$, $v(\alpha, \beta, t)$, $w(\alpha, \beta, t)$ are projections of the displacements of the middle surface points on α, β, z, respectively; $\partial\Lambda$ is a clamped part of the shell boundary, n is normal to $\partial\Lambda$.

The Rayleigh–Ritz method is used to calculate the eigenfrequencies and the eigenmodes of linear vibrations. In this case only geometric boundary conditions are satisfied. It is assumed that the squares of rotation angles of the middle surface normal are significantly less than unit. The potential energy of the shell has the following form [22,23]:

$$\begin{aligned} \Pi = \frac{E}{2(1+\mu)} \int_\Lambda \Bigg\{ & \frac{1}{1-\mu} \Big[(\varepsilon_{11}^2 + 2\mu\varepsilon_{11}\varepsilon_{22} + \varepsilon_{22}^2) h(\alpha,\beta) \\ & + \frac{h^3(\alpha,\beta)}{12} (\chi_1^2 + 2\mu\chi_1\chi_2 + \chi_2^2) \Big] \\ & + \frac{1}{2} \left(\varepsilon_{12}^2 h(\alpha,\beta) + \frac{1}{3}\kappa^2 h^3(\alpha,\beta) \right) \Bigg\} AB \, d\alpha \, d\beta, \end{aligned} \tag{2}$$

$$\begin{aligned} &\varepsilon_{11} = \varepsilon_1 + \frac{1}{2}\left(-\frac{w_\alpha}{A}\right)^2; \quad \varepsilon_{22} = \varepsilon_2 + \frac{1}{2}\left(-\frac{w_\beta}{B}\right)^2; \quad \varepsilon_{12} = \gamma + \frac{w_\alpha w_\beta}{AB}; \\ &\varepsilon_1 = \frac{u_\alpha}{A} + \frac{A_\beta}{AB} v - k_1 w; \quad \varepsilon_2 = \frac{v_\beta}{B} + \frac{B_\alpha}{AB} u - k_2 w; \quad \gamma = \frac{B}{A}\left(\frac{v}{B}\right)_\alpha + \frac{A}{B}\left(\frac{u}{A}\right)_\beta; \\ &\chi_1 = \frac{1}{A}\left(\frac{w_\alpha}{A}\right)_\alpha + \frac{A_\beta}{AB}\frac{w_\beta}{B}; \quad \chi_2 = \frac{1}{B}\left(\frac{w_\beta}{B}\right)_\beta + \frac{B_\alpha}{AB}\frac{w_\alpha}{A}; \\ &\kappa = \frac{1}{B}\left(\frac{w_\alpha}{A}\right)_\beta - \frac{B_\alpha}{AB}\frac{w_\beta}{B} = \frac{1}{A}\left(\frac{w_\beta}{B}\right)_\alpha - \frac{A_\beta}{AB}\frac{w_\alpha}{A}, \end{aligned} \tag{3}$$

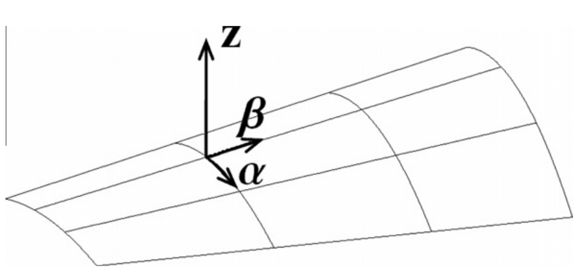

Fig. 1. Sketch of the middle surface of the shallow shell.

where ε_{11}, ε_{22}, ε_{12} are middle surface strains; χ_1, χ_2, κ are curvatures and torsion of the middle surface; $h(\alpha, \beta)$ is the variable thickness of the shell; $A = \sqrt{g_{11}}$ and $B = \sqrt{g_{22}}$ are Lame parameters; g_{11} and g_{22} are the coefficients of the first quadratic form of the middle surface; $(\ )_\alpha$ and $(\ )_\beta$ are denoted differentiation with respect to α and β, respectively. The integral is taken over the area of the shell base Λ. In this section the linear vibrations are treated and the nonlinear terms are not taken into account in expressions (3). The kinetic energy of the shell is the following:

$$T = \frac{\rho}{2} \int_\Lambda (\dot{w}^2 + \dot{u}^2 + \dot{v}^2) h(\alpha, \beta) AB \, d\alpha \, d\beta, \tag{4}$$

where ρ is a density of a shell material.

Linear periodic vibrations are presented as

$$\begin{aligned} u(\alpha, \beta, t) &= U(\alpha, \beta)\exp(i\omega t); \\ v(\alpha, \beta, t) &= V(\alpha, \beta)\exp(i\omega t); \\ w(\alpha, \beta, t) &= W(\alpha, \beta)\exp(i\omega t). \end{aligned}$$

The minimum value of the Lagrange functional $L = T - \Pi$ is determined to obtain the parameters of the shell linear vibrations. In order to obtain this minimum value the eigenmodes of the shell vibrations are expanded into the truncated series of trial functions in the following way:

$$\begin{aligned} W(\alpha, \beta) &= \sum_{i=1}^{N_1} a_i w_i(\alpha, \beta); \\ U(\alpha, \beta) &= \sum_{i=N_1+1}^{N_2} a_i u_i(\alpha, \beta) \\ V(\alpha, \beta) &= \sum_{i=N_2+1}^{N_3} a_i v_i(\alpha, \beta), \end{aligned} \tag{5}$$

where $w_i(\alpha, \beta)$, $u_i(\alpha, \beta)$, $v_i(\alpha, \beta)$ are trial functions. Eqs. (5) are substituted into the Lagrange functional and the following function is obtained:

$$L = L(a_1, \ldots, a_{N_3}). \tag{6}$$

The determination of the minimum of the Lagrange functional is reduced to the following equations: $\partial L/\partial a_j = 0$; $j = 1, \ldots, N_3$. These equations can be presented in the form of the eigenvalue problem:

$$(K - \omega^2 C)\widetilde{A} = 0, \tag{7}$$

where $\widetilde{A} = (a_1, \ldots, a_{N_3})$.

3. Linear vibrations of the shell interacting with a fluid

The free vibrations of the shallow shells in perfect incompressible fluid are considered. The pressure difference is determined to calculate the eigenfrequencies and eigenmodes of linear vibrations. It is assumed that the fluid motion is irrotational and the velocity potential Φ satisfies the Laplace equation

$$\Delta\Phi(\mathbf{x}, t) = 0,$$

where $\mathbf{x}$ is spatial coordinate vector. The boundary conditions on the shell surfaces S have the following form:

$$\left.\frac{\partial\Phi(\mathbf{x}, t)}{\partial\mathbf{n}}\right|_{S^\pm} = \left.\frac{\partial w(\mathbf{x}, t)}{\partial t}\right|_{S^\pm}.$$

If $||\mathbf{x}|| \to \infty$, the velocities of fluid motions tend to zero:

$$\lim_{||\mathbf{x}|| \to \infty} \nabla\Phi(\mathbf{x}, t) = 0.$$

This expression is called the Sommerfeld condition [14].

The equations of the vibrations of the shell interacting with a fluid can be presented in the following operational form: