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ESA formulation for large displacement analysis of framed structures with elastic–plasticity

Goran Turkalj [a,*], Josip Brnic [a], Jasna Prpic-Orsic [b]

[a] Department of Engineering Mechanics, Faculty of Engineering, University of Rijeka, Vukovarska 58, HR-51000 Rijeka, Republic of Croatia

[b] Department of Naval Architecture, Faculty of Engineering, University of Rijeka, Vukovarska 58, HR-51000 Rijeka, Republic of Croatia

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Abstract

In this work, a spatial beam element for geometrically and materially non-linear analysis of framed structures is presented in this work. The equilibrium equations of a straight beam element are formulated using an updated Lagrangian (UL) incremental description. Internal moments are represented as the resultants of stresses calculated by engineering theories: Euler–Bernoulli–Navier theory for bending and Saint-Venant theory for torsion. Although the element developed can undergo large displacements and rotations, strains are assumed to be small. The non-linear cross-sectional displacement field including large rotation effects is introduced in the analysis, resulting in the geometric potential of bending and torsional moments which corresponds to that of semitangential behaviour. In such a way, the joint equilibrium of non-collinear elements is provided. For the force recovering, the external stiffness approach (ESA) is presented as an alternative to the common natural deformation approach (NDA). Material non-linearity is introduced for an elastic–perfectly plastic material through the plastic hinge formation at finite element nodes and for this a new plastic reduction matrix of the element is determined. The interaction of element forces at a hinge and the possibility of elastic unloading are taken into account. The effectiveness of the numerical algorithm discussed is validated through the test problem.

Keywords: Framed structures; Beam element; Elastic–plasticity; Non-linear displacement field; Updated lagrangian formulation; External stiffness approach; Plastic hinges

1. Introduction

In the field of structural engineering beams and frames constitute a very important class of load-carrying components, where they are applied both in their stand-alone forms and as stiffeners for some plate or shell assemblages [1,2]. Because such structures, especially those of thin-walled cross-sections, could display very complex structural behaviour under a large displacement and rotation regime [3], the development of advanced non-linear beam models, which comprise both geometric and material inelasticity, has been a major activity of many structural engineering researchers in the past years [4–8].

* Corresponding author. Tel.: +385 51 651 499; fax: +385 51 651 490.
E-mail address: turkalj@rijeka.riteh.hr (G. Turkalj).

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In the non-linear finite element analysis, a response of a load-carrying structure is usually solved using some of incremental descriptions like the total and updated Lagrangian ones, respectively, or the Eulerian description [9,11]. Each description utilizes a different structural configuration for system quantities referring and results in the form of a set of non-linear equilibrium equations of the structure. This set can further be linearized and should be solved using some incremental-iterative scheme, which consists of three main phases. The first or predictor phase comprises evaluating the overall structural stiffness and solving for the displacement increments from the approximated incremental equilibrium equations for the structure. Using the standard transformation process displacement increments of each finite element can be determined immediately. The second or corrector phase involves the geometry updating of each finite element and the determination of element nodal forces using some force recovery algorithm. The third or checking phase comprises checking if the adopted convergence criterion of iteration is achieved in the current increment by comparing with the pre-set tolerance value.

The first part of the present work comprises only geometrical non-linearities of an elastic, straight and prismatic beam member with solid and doubly symmetric cross-sections. Displacements and rotations are allowed to be large but strains are small. Loading of a considering structure is assumed to be static and conservative, while internal moments are represented as the resultants of stresses calculated by engineering theories. The element geometric stiffness is derived using the updated Lagrangian description and the non-linear displacement field of a beam cross-section, which includes non-linear displacement terms due to three-dimensional rotation effects. In such a way, the incremental geometric potential corresponding to the semitangential bending and torsional moments is obtained, ensuring thus the moment equilibrium conditions at a joint of beam members with different space orientations [12]. For the force recovering, the natural deformation approach (NDA) is frequently used in the updated Lagrangian (UL) incremental formulation, but its application requires a finite element to pass the rigid-body test [13]. If this test is not passed, the NDA cannot serve as the force recovery algorithm and, instead, some alternative should be available and thus, the external stiffness approach (ESA) is proposed in this work. At this approach, the external stiffness matrix of a beam element is included in the force recovering to exclude the rigid-body effects [14].

Introducing the possibility of elastic–perfectly plastic material behaviour, the analysis is further enhanced to the elastic–plastic material behaviour. Although the plastic-zone model is usually considered as the 'exact model' because it explicitly accounts for the spread of plasticity throughout the beam member [15], in this work, because of the computational advantages, the plastic hinge model is applied [16]. In such an elastic–plastic model, it is assumed that all plastic effects, when occur, are concentrated in the zero-length plastic hinges at the finite element ends, while the element between hinges remains linear-elastic. Supposing the existence of a single-function yield surface in terms of the beam stress-resultants obtained by the ESA and using the normality principle, a plastic reduction matrix of the beam element is consistently derived, a function of which is to keep the element incremental forces at a plastic hinge to move tangentially to the yield surface [17]. Since at the end of that increment the yield criterion is violated, the return of element nodal forces at a plastic hinge to the yield surface should be performed. The abovementioned numerical algorithm is implemented into a computer program and the effectiveness is validated through the test problem.

2. Basic considerations for solid beam

2.1. Kinematics of beam

The deformation of an initially straight beam with undeformable cross-section is studied. A right-handed Cartesian co-ordinate system (z,x,y) is chosen in such a way that z-axis coincides with the beam axis passing through the centroid O of each cross-section, while x- and y-axes are the principal inertial axes of the cross-section. Incremental displacement measures of a cross-section are defined as

$$w_o = w_o(z), \quad u_o = u_o(z), \quad v_o = v_o(z),$$
$$\varphi_z = \varphi_z(z) \quad \varphi_x = -\frac{\mathrm{d}v_o}{\mathrm{d}z} = \varphi_x(z),$$
$$\varphi_y = \frac{\mathrm{d}u_o}{\mathrm{d}z} = \varphi_y(z) \tag{1}$$

where w_o, u_o and v_o are rigid-body translations of the cross-section in the z-, x- and y-direction at the centroid, respectively; φ_z, φ_x and φ_y are rigid-body rotations about the z-, x- and y-axis, respectively.

Let $\boldsymbol{r}_0$ denotes the position vector of a material point in the reference configuration and $\mathbf{U}_o$ the translation displacement vector of the centroid. If the assumption of small rotations is valid, then the displacement vector $\mathbf{U}_{\mathrm{ldf}}$, representing the linear displacement field of a cross-section, can be written in the following form:

$$\mathbf{U}_{\mathrm{ldf}} = \mathbf{U}_o + \tilde{\varphi}\mathbf{r}_0 \tag{2}$$

where