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Large amplitude vibrations of anisotropic cylindrical shells

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Abstract

A semi-analytical method is developed in conjunction with shearable shell theory and modal expansion approach to predict the influence of geometrical non-linearities on free vibrations of anisotropic laminated cylindrical shells. Shear deformation and rotary inertia effects are taken into account in the equations of motion. The hybrid method developed in this theory is a combination of classical finite element approach, shearable shell theory and modal coefficient procedure. The displacement functions are obtained by the *exact* solution of the equilibrium equations of anisotropic cylindrical shells and thereafter, the mass and linear stiffness matrices are derived by *exact analytical integration*. Green exact strain–displacement relations are used to obtain the modal coefficients for these displacement functions. The second- and third-order non-linear stiffness matrices are then calculated by *precise analytical integration* and superimposed on the linear part of equations to establish the non-linear modal equations. The linear and non-linear natural frequency variations are determined as a function of shell parameters for different cases. The comparison shows that the numerical analysis is of good reliability on the prediction of the experimental results.

Keywords: Non-linear; Vibrations; Anisotropic shells; Shear deformations

1. Introduction

The design of missiles and launch vehicles, nuclear components and shipbuilding structures makes extensive use of composite cylindrical shells. The reliability of these structures depends directly on their component performance. Therefore, the dynamic behaviour of these structures is of keen interest in the design of structural elements in the space and aeronautical, petroleum and nuclear industries. These cylindrical structures often experience large amplitude vibrations, which are greater than the shell thickness. This problem has given rise to a number of studies of the non-linear vibrations of cylindrical shells subjected to different loads. In addition, due to the high ratio of tangential Young's modulus to transverse shear modulus in composite materials such as graphite–epoxy and boron–epoxy, the shear deformation effect on the non-linear behaviour of anisotropic composite shells is more significant than that of isotropic ones. This effect plays a very important role in reducing the effective flexural stiffness of anisotropic composite plates and shells.

Many works exist in the literature concerning non-linear models for isotropic [1–3] and anisotropic shell analysis [4–7]. Also, a number of theories for layered

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Nomenclature

A, B, C, D, E defined by Eq. (2)
$AA_{jk}, BB_{jk}, \ldots, TT_{jk}$ modal coefficients determined by Eq. (18)
$AA_{ijk}, BB_{ijk}, \ldots, TT_{ijk}$ modal coefficients determined by Eq. (16)
$AA_{ijks}, BB_{ijks}, \ldots, TT_{ijks}$ modal coefficients determined by Eq. (17)
$AUX^{(1)}_{ijk}, \ldots, AUX^{(58)}_{ijk}$ modal coefficients determined by Eq. (16)
$AUX^{(1)}_{ijks}, \ldots, AUX^{(58)}_{ijks}$ modal coefficients determined by Eq. (17)
$f_i (i = 1\text{–}10)$ coefficients of determinant of the matrix $[H]$, Eq. (3)
$k_{ij}{}^{L}$ general element of linear stiffness matrix, Eq. (15)
k^{NL2}_{ijk} general element of second-order non-linear stiffness matrix, Eq. (15)
k^{NL3}_{ijks} general element of third-order non-linear stiffness matrix, Eq. (15)
L length of shell
L_i equations of motion, Eq. (1)
m axial mode number
$\overline{m}$ defined by $\frac{m\pi}{L}$
m_{ij} general element of mass matrix, Eq. (15)
M_x, M_θ, $M_{x\theta}$, $M_{\theta x}$ moment resultants
n circumferential wave number
N_x, N_θ, $N_{x\theta}$, $N_{\theta x}$ in-plane force resultants
P_{ij} terms of elasticity matrix, Eq. (1)
Q_{xx}, $Q_{\theta\theta}$ the transverse force resultants, Eq. (8)
R mean radius of the shell
t thickness of the shell
u, v, w axial, circumferential and radial displacement respectively
U_m, V_m, W_m, β_{xm}, $\beta_{\theta m}$ amplitudes of u, v, w, β_x, and β_θ associated with m_{th} axial mode number
x axial coordinate
α_i, β_i, γ_i and δ_i defined by Eq. (4)
β_x and β_θ the rotations of the normal about the coordinates of the reference surface
η_i complex roots of the characteristic, Eq. (3)
ε_L linear deformation vector, Eq. (7)
ε_{NL} non-linear deformation vector, Eq. (11)
ε^0_x and ε^0_θ normal strains of the reference surface
γ^0_x and γ^0_θ in-plane shearing strains of the reference surface
κ_x and κ_θ change in the curvature of the reference surface
τ_x and τ_θ torsion of the reference surface
μ^0_x and μ^0_θ shearing strains
θ circumferential coordinate
ϕ_T angle for the whole open shell
ρ density of the shell material
Γ_i vibration amplitude
ψ_i defined in Eq. (27)

List of matrices:
$[A]_{(10\times10)}$ defined by Eq. (5)
$[B]_{(10\times10)}$ defined by Eq. (7)
$[H]_{(5\times5)}$ defined by Eq. (3)
$[k^{(L)}]$ linear stiffness matrix, Eq. (9)
$[k^{(NL2)}]$ second-order stiffness matrix, Eq. (16)
$[k^{(NL3)}]$ third-order stiffness matrix, Eq. (17)
$[m]$ mass matrix, Eq. (9)
$[N]$ shape function matrix, Eq. (6)
$[P]$ elasticity matrix, Eq. (8)
$\{C\}$ vector for arbitrary constants, Eq. (5)
$\{q\}$ time-related vector, Eq. (10)
$\{\delta_i\}$ degrees of freedom at node i
$\{\varepsilon_L\}$ and $\{\varepsilon_{NL}\}$ linear and non-linear deformation vector, Eq. (11)

anisotropic shells exist [8], which are developed for thin shells and are based on the Kirchhoff–Love hypothesis. Most of these approaches can include various degrees of non-linearity in the strain–displacement relations in representing the displacement and rotations. A more rigorous study of non-linear free flexural vibrations of circular cylindrical shells was conducted by Atluri [9] who compared his results with the available data and concluded the possibility of the softening type of non-linearity. A set of non-linear strain–displacement relations for axisymmetric thin shells, based on the Kirchhoff assumptions, subject to large displacements with moderate rotations by retaining more terms are given in [10]. This work is based on the Kirchhoff assumptions. It is shown that non-linear strains arising from products of in-plane strain terms, which were omitted in some theories, may be important in certain buckling problems.

An overall concept of the non-linear analysis of shell structures is developed in [11]. This monograph outlines a survey of theories for the analysis of plates and shells with small deflections that then lead to the theory of shells undergoing large deflections and rotations applicable to elastic laminated anisotropic shells. For a recent survey and discussions about the perspectives in non-linear vibrations of shells see [12–14] that contain extensive