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On the effect of twist angle on nonlinear galloping of suspended cables

Angelo Luongo [a,*], Daniele Zulli [a], Giuseppe Piccardo [b]

[a] DISAT – University of L'Aquila, 67040 L'Aquila, Italy
[b] DICAT – University of Genoa, 16145 Genova, Italy

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ABSTRACT

A nonlinear model of cable, able to twist, is formulated. For small sag-to-length ratios (e.g. 1/10) and technical parameter values proper to electrical transmission lines, the motion is ruled by the classical equations of the perfectly flexible cable, plus a further equation governing the twist evolution. A two degree-of-freedom system is successively obtained via a Galerkin procedure. The relevant nonlinear ODE's are dealt with a Multiple Scale approach, under 2:1 internal resonance condition and no resonance conditions, in order to investigate Hopf bifurcations and post-critical behaviors. All the numerical results are compared with those furnished by the flexible model, and the influence of twist is discussed.

1. Introduction

It is well known that the aerodynamic forces acting on a non-circular rigid cylinder, subjected to a wind flow, depend, besides on the mean wind velocity, on the exposure to the flow of the body, that is on the *attitude* of the cylinder cross-section. When an elastic beam is analyzed in the framework of the quasi-steady theory of the aerodynamic forces, the loads are usually evaluated referring to the *initial attitude* of the section (see [1]), and rotations taken into account only to determine, in an approximate manner (see [2]), the fluid to structure relative velocity. A more refined analysis, however, is possible, in which the time-dependent *actual attitude* of the cross-section is considered, as described by the so-called *twist angle* (i.e. by the rotation of the section around its normal axis). In contrast, when a pretensioned string is studied, such an angle is usually not included among the kinematic descriptors of the body, since all the rotations are believed to be unimportant in capturing the main structural behavior. Therefore, a model of perfectly flexible string is adopted (one-dimensional not polar continuum), and the aerodynamic forces evaluated with reference to the initial attitude of the section, which is assumed to remain immutable in time. The problem is made even more complicated when a sagged cable is considered. Indeed, due to the steady part of the aerodynamic forces and to the high flexibility of the structure, the cable significantly changes its equilibrium configuration and, therefore, its exposure to the flow. Hence, in addition to a dynamic rotation, a static velocity-dependent rotation of the section must be considered in evaluating the aerodynamic forces.

The aeroelastic instability of sagged cables has been widely studied in the literature. Luongo and Piccardo [3] have studied the nonlinear galloping of cables in 2:1 internal resonance condition, by using a perfectly flexible cable model [4,5] and accounting for the static rotation only. In a successive work [6], they have tentatively corrected the classical cable model to account for the twist, by using a quite simplified model. Yu et al. [7], McConnel and Chang [8], White et al. [9] have employed a model of cable-beam, accounting for twisting but not for bending, and neglecting the cable initial curvature in defining the torsion strain. In contrast, they have considered a realistically coupled extension–torsion constitutive law, based on experimental results. Recently, Luongo et al. [10], have formulated a consistent linear model of cable-beam accounting for the (small) curvature of the cable, as well as for bending and torsional stiffness. By retaining only the leading terms in each equations, they obtained linear reduced equations, amenable to an analytical solution, identical to that of the perfectly flexible model, plus an additional equation accounting for both bending and torcent moments. The model permitted to detect the influence of the dynamic twist on the critical wind velocity.

In this paper, the model presented in [10] is reformulated in the nonlinear range and nonlinear, reduced equations are derived along the same lines. These equations are obviously a particular case of more complete models, as, for example, that of Lu and Perkins [11]; these models, however, are composed by very complex equations and suffer of some numerical problems related to the

* Corresponding author.
E-mail addresses: luongo@ing.univaq.it (A. Luongo), danzulli@ing.univaq.it (D. Zulli), giuseppe.piccardo@unige.it (G. Piccardo).

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existence of boundary layers, caused by the smallness of the flexural terms (nearly-singular equations). A simple two-degree-of-freedom nonlinear system is then derived from the continuous model via a Galerkin procedure, and the critical and post-critical aeroelastic behavior of the cable investigated in resonant and non-resonant cases via a Multiple Scale perturbation approach (see [12]). The role played by the dynamic twist is finally highlighted.

The paper is organized as follows. The reduced equations of motion are formulated in Section 2. The discretization is performed in Section 3. The perturbation analysis is carried out in Section 4, where the amplitude modulation equations are derived. These latter are numerically studied in Section 5 for some sample systems. Finally, some conclusions are drawn in Section 6.

2. Model

The cable is modeled as a body made of a flexible centerline and rigid cross-sections restrained to remain orthogonal to the axis (shear-undeformable beam). It is assumed to be uniformly iced and loaded by a wind flow of mean velocity $\mathbf{U} = U\mathbf{a}_z$, blowing horizontally. Three different configurations are considered, described in the following (Fig. 1). (a) The *initial configuration* $\mathscr{C}_0$, taken by the body at the time $t = 0$, under the action of its self-weight $-mg\mathbf{a}_y$ (including the ice accretion). This configuration is planar, and belongs to the vertical $(\mathbf{a}_x, \mathbf{a}_y)$-plane. The cable is prestressed in $\mathscr{C}_0$ by an axial internal force $T_0(s_0)$, depending on the (unstretched) abscissa s_0; other internal forces (shear, bending and torcent moments) are neglected. (b) The *reference configuration* $\overline{\mathscr{C}}$, assumed by the body at the time $t = 0^+$, in which *static* wind forces $\bar{\mathbf{b}}_a(s)$ (with the stretched abscissa $s \simeq s_0$) act on the cable. Under the simplifying hypothesis that $\bar{\mathbf{b}}_a$ is uniform on the length of the cable, $\overline{\mathscr{C}}$ still lies in a plane, forming an angle φ with the vertical plane. If shear and internal moments are still neglected, equilibrium requires that the resultant force $\bar{\mathbf{b}} := \bar{\mathbf{b}}_a - mg\mathbf{a}_y$ lies in the plane of the cable. By vanishing its component along the binormal direction $\bar{\mathbf{a}}_3$, it follows that:

$$\bar{b}_{a_3}(\varphi, U) + mg\sin\varphi = 0 \tag{1}$$

being $\bar{b}_{a_3} = \bar{\mathbf{b}}_a \cdot \bar{\mathbf{a}}_3$, $\mathbf{a}_y \cdot \bar{\mathbf{a}}_3 = -\sin\varphi$, and where the dependence on the static aerodynamic force $\bar{b}_{a_3}$ on the configuration variable φ and on the flow velocity U has been made explicit. Eq. (1) implicitly defines the nonlinear, non-trivial equilibrium path $\varphi = \varphi(U)$. In this wind-dependent reference configuration $\overline{\mathscr{C}}$, the cable is prestressed by an axial internal force $\overline{T}(s)$, modified with respect to $\mathscr{C}_0$ by the static aerodynamic forces. Due to this circumstance, also the natural frequencies and modes of the cable are modified. (c) The *actual configuration* $\mathscr{C}$, assumed by the cable at the time $t > 0$, in which the body is loaded also by (non-uniform) dynamic wind forces $\mathbf{b}_a - \bar{\mathbf{b}}_a$, depending on displacement and velocity of the cable. In this configuration $\mathscr{C}$, all the internal forces and moments are considered to contribute to the dynamic equilibrium of the body.

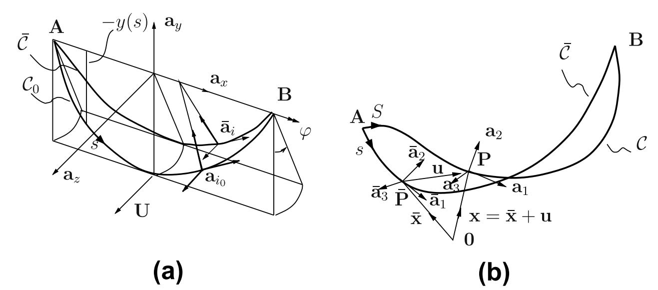

Fig. 1. Cable configurations: (a) initial $\mathscr{C}_0$ and reference $\overline{\mathscr{C}}$ configurations; (b) actual configuration $\mathscr{C}$.

The equations of motion governing the dynamics of the cable, referred to the configuration $\overline{\mathscr{C}}$, are derived in the following. The mechanical model and the aerodynamic model are formulated separately.

2.1. Mechanical model

The reference configuration $\overline{\mathscr{C}}$ is described by the planar curve $\bar{\mathbf{x}} = \bar{\mathbf{x}}(s)$ and by the cross-section inertial principal triad $\bar{\boldsymbol{\beta}} := \{\bar{\mathbf{a}}_1(s,t), \bar{\mathbf{a}}_2(s,t), \bar{\mathbf{a}}_3(s,t)\}$, assumed to be coincident with the Frenet triad (Fig. 1b). Here, $\bar{\mathbf{a}}_1 \equiv \bar{\mathbf{x}}'$ is the tangent, $\bar{\mathbf{a}}_2$ the normal and $\bar{\mathbf{a}}_3$ the binormal to the curve, the dash denoting s-differentiation. Therefore, $\bar{\mathbf{a}}_1' = \bar{\kappa}\bar{\mathbf{a}}_2$, $\bar{\mathbf{a}}_2' = -\bar{\kappa}\bar{\mathbf{a}}_1$, $\bar{\mathbf{a}}_3' = \mathbf{0}$, with $\bar{\kappa} = \bar{\kappa}(s)$ the curvature in $\overline{\mathscr{C}}$.

The actual configuration of the body is described by the nonplanar curve $\mathbf{x} = \mathbf{x}(s,t)$ and the inertial principal triad $\boldsymbol{\beta} := \{\mathbf{a}_1(s,t), \mathbf{a}_2(s,t), \mathbf{a}_3(s,t)\}$.

The transport is described by the displacement vector field $\mathbf{u}(s,t)$ and the rotation tensorial field $\mathbf{R}(s,t)$, which leads the triad $\bar{\boldsymbol{\beta}}$ to match the triad $\boldsymbol{\beta}$:

$$\mathbf{x} = \bar{\mathbf{x}} + \mathbf{u}, \qquad \mathbf{a}_i = \mathbf{R}\bar{\mathbf{a}}_i, \quad i = 1,2,3. \tag{2}$$

The scalar representation of $\mathbf{R}$, involving three elementary rotations ϑ_i, is given in [13] and in many other papers. The shear-undeformability constraints require that, in the actual configuration, the tangent $\mathbf{x}'$ to the centerline is parallel to the normal $\mathbf{a}_1$ with regard to the cross-section, namely, $\mathbf{x}' = (1+\varepsilon)\mathbf{a}_1$, where ε is the axial strain. From this condition, the strain ε and the rotations ϑ_2 and ϑ_3 are derived as functions of four independent configuration variables, the three components u, v and w of vector $\mathbf{u}$ on $\bar{\boldsymbol{\beta}}$, and the *twist angle* $\vartheta := \vartheta_1$. Then, the incremental *bendings* $\hat{\kappa}_2, \hat{\kappa}_3$ and *torsion* $\hat{\kappa}_1$ are introduced as independent components on $\bar{\boldsymbol{\beta}}$ of the skew-symmetric tensor:

$$\widehat{\mathbf{K}} = \mathbf{R}^{\mathrm{T}}\mathbf{R}'. \tag{3}$$

The following strain measures, expanded up to second-order terms, are obtained [13]:

$$\begin{aligned}
\varepsilon &= u' - \bar{\kappa}v + \frac{1}{2}[(v' + \bar{\kappa}u)^2 + w'],\\
\hat{\kappa}_1 &= \vartheta' + \bar{\kappa}w' + \bar{\kappa}^2 vw' + w'v'' + \bar{\kappa}'uw',\\
\hat{\kappa}_2 &= -w'' + \bar{\kappa}\vartheta + [(u' - \bar{\kappa}v)w']' + \vartheta[(\bar{\kappa}u)' + v''],\\
\hat{\kappa}_3 &= v'' + (\bar{\kappa}u)' + \vartheta w'' - \frac{1}{2}\bar{\kappa}(\vartheta^2 + w'^2) - [(\bar{\kappa}u + v')(u' - \bar{\kappa}v)]'.
\end{aligned} \tag{4}$$

The equations of motion are derived via the extended Hamilton principle (see, e.g. [14]):

$$\begin{aligned}
\delta H := \int_{t_1}^{t_2}\int_0^{\ell} \{ & m(\dot{u}\,\delta\dot{u} + \dot{v}\,\delta\dot{v} + \dot{w}\,\delta\dot{w}) + \mathscr{J}_1\dot{\vartheta}\delta\dot{\vartheta} + (b_1 - c_u\dot{u})\delta u\\
& + (b_2 - c_v\dot{v})\delta v + (b_3 - c_w\dot{w})\delta w + (c_1 - c_\vartheta\dot{\vartheta})\delta\vartheta + EA\varepsilon\delta\varepsilon\\
& + GJ\hat{\kappa}_1\delta\hat{\kappa}_1 + EI_2\hat{\kappa}_2\delta\hat{\kappa}_2 + EI_3\hat{\kappa}_3\delta\hat{\kappa}_3 - \overline{T}\varepsilon_{\mathrm{II}}\delta\varepsilon_{\mathrm{II}}\}\mathrm{d}s\,\mathrm{d}t = 0\\
& \forall\delta u, \delta v, \delta w, \delta\vartheta,
\end{aligned} \tag{5}$$

where ℓ is the cable length; EA, GJ, EI_2 and EI_3 are the axial, torsional and bending stiffnesses, respectively; b_1, b_2, b_3 and c_1 are the external forces and couple densities; c_u, c_v, c_w and c_ϑ are the structural damping coefficients; $\varepsilon_{\mathrm{II}}$ is the second-order part of the axial strain, accounting for the prestress working; m is the mass linear density and I_1 is the inertia polar moment of the section. It should be noted that an uncoupled constitutive law among the incremental internal forces and the incremental strains has been assumed in Eq. (5). By substituting Eq. (4) in Eq. (5), performing the variations and integrating by parts, a set of four differential equations in the independent configuration variables is drawn. By enforcing the constraint